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Report of the Meeting and Exhibition commemorating the bi-centenary of the death of Sir Isaac Newton.

November 25-26, 1927.

The fourth annual meeting of The History of Science Society was held in pursuance of an act of its Council of last year, to commemorate the bicentenary of the death of Sir ISAAC NEWTON. In view of this unusual occasion, and of the manifold interests NEWTON had, it was therefore considered expedient to ask those societies most interested to cooperate by appointing representatives on the Committee preparing the Program. The names of those who served follow :

COMMITTEE ON PROGRAM.

Dr. DAVID EUGENE SMITH, Columbia University, Chairman Dr. HENRY CREW, Northwestern University	The History of Science Society
Dr. R. C. ARCHIBALD, Brown University Dr. E. W. BROWN, Yale University Dr. FLORIAN CAJORI, University of California	The American Mathematical Society, and The Mathematical Association of America
Dr. A. O. LEUSCHNER, University of California Dr. FREDERICK H. SEARES, Mount Wilson Observatory	The American Astronomical Society
Dr. LEIGH PAGE, Yale University	The American Physical Society.
Mr. FREDERICK E. BRASCH, Library of Congress	Secretary to the Committee.

COMMITTEE ON ARRANGEMENTS

Dr. LAO G. SIMONS, Chairman, Hunter College, N. Y.
Dr. VERA SANFORD, The Lincoln School, N. Y.
Miss FRANCES M. CLARKE, Teachers College, N. Y.
Miss HELEN WALKER, Teachers College, N. Y.
Dr. LESTER S. HILL, Hunter College, N. Y.
Mr. JOHN A. SWENSON, Wadleigh High School, N. Y.

The meeting was called to order by the President of The History of Science Society, Dr. DAVID EUGENE SMITH, at 10.00 o'clock A. M. Friday, November 25, 1927, in the Educational Hall of the American Museum of Natural History, New York City.

The two days' program consisted entirely of papers on various phases of Sir ISAAC NEWTON's contributions to Astronomy, Mathematics, Physics, Chemistry, Theology, problems of the Mint, and the development of science since his day; also some indication of NEWTON's influence on the early science in the American Colonies from 1687 to 1779. Two papers were devoted to each subject and these were given by scholars of distinction — from both the United States and Canada. These addresses have been published in a memorial volume by WILLIAMS and WILKINS, Baltimore, for The History of Science Society.

The object of this American Commemoration was two-fold, first to honor the great name of NEWTON, and second to acquaint the younger generation of students, as well as the public at large, with the contributions made in the various fields of knowledge in which NEWTON laboured. From the standpoint of interest and attendance, the meeting was successful. Similar celebrations have taken place in other cities. Particularly we note the program given in Grantham, NEWTON's birthplace, on Saturday March 20, 1927, the exact date of the bicentenary of his death, which was attended by many of the greatest scholars of the British Empire, as well as by representatives from abroad.

Inasmuch as the full text of these valuable lectures will be preserved in the Memorial Volume above mentioned, it will suffice to place here on record the names of the lecturers and the titles of their papers.

First sitting. November 25, 1927. 10 o'clock A. M.

Dr. DAVID EUGENE SMITH, Columbia University. Introductory Address on NEWTON's life and work.

Dr. D. C. MILLER, Case School of Applied Science. NEWTON and optics.

Dr. G. D. BIRKHOFF, Harvard University. NEWTON's philosophy of gravitation with special reference to modern relativity ideas.

Dr. W. W. CAMPBELL, President, University of California. NEWTON's influence on the development of astrophysics.

Second sitting. Same day. 2 o'clock P. M.

Dr. M. I. PUPIN, Columbia University. NEWTON and dynamics.

Dr. PAUL R. HEYL, U. S. Bureau of Standards. NEWTON as an experimenter.

Dr. E. W. BROWN, Yale University. Developments following from NEWTON's work.

Dr. FLORIAN CAJORI, University of California. NEWTON's twenty years delay in announcing the law of gravitation. NEWTON's early study of the Apocalypse. (Dr. CAJORI was not present).

Third sitting. November 26. 10 o'clock A. M.

Dr. LYMAN C. NEWELL, Boston University. NEWTON's work in alchemy and chemistry.

Dr. GEORGE S. BRETT, Toronto University. NEWTON's place in the history of religious thought.

GEORGE E. ROBERTS, Esq., The National City Bank of New York. NEWTON in the Mint.

FREDERICK E. BRASCH, Esq., Library of Congress. NEWTON's first disciple in the American colonies.

Newton Exhibition.

The great exhibition of books, portrait prints, letters, autographed documents and medallic illustrations of Sir ISAAC NEWTON, and of his mathematical contemporaries, which supplemented the addresses, were all arranged in suitable cases on three sides of the hall in which the audience gathered. This most imposing collection probably will never be duplicated at any one time and in itself constituted one of the main attractions of the meeting.

The excellent arrangement of these beautiful memorials was carried on under the personal direction of Dr. DAVID EUGENE SMITH. From Dr. SMITH's private collection came 125 of the most beautiful portrait prints of NEWTON, of various ages and painters; also 25 medals of about that many engravers, and a large assortment of letters and autographed documents. Dr. SMITH loaned also over 40 various editions of NEWTON's *Principia*, *Opticks*, *Fluxions*, *Universal Arithmetics*, as well as essays on NEWTON's life and work.

Besides the NEWTON material from this same private collection, came portraits and autographed letters of HALLEY, BARROW, COTES, WREN, WALLIS, HUYGENS, CASSINI, all friends and coworkers of NEWTON. There were also exhibited portraits of FLAMSTEED, LEIBNIZ, DESCARTES, the BERNOULLIS, WOLF, GAUSS, LA PLACE, GASSENDI, PASCAL, KEPLER, GALILEO and COPERNICUS. Most interesting probably was a collection of letters from LEIBNIZ to NEWTON with notes in NEWTON's handwriting. This copy of bound letters was from NEWTON's private library. Another collection of letters to NEWTON came from four generations of Astronomers Royal of France, the CASSINI family, to whom NEWTON was indebted for his data on the size of the earth. From the Babson Institute, Wellesley, came also a magnificent collection of 36 items representing the various editions of NEWTON's works, the most interesting and valuable being a copy of the first edition of the *Principia Mathematica*, 1687, containing notes and corrections in NEWTON's own handwriting. There was also a copy of NEWTON's *Opticks*, 2nd edition, with notes in his handwriting. All these contained NEWTON's bookplate and autograph, showing they were from his private library. Besides a collection of portrait prints of NEWTON, a cast of the sundial as well as of his inscription cut on the window sill of the Grantham grammar school, made by the boy NEWTON.

From Mr. GEORGE A. PLIMPTON's (New York) great collection of Rare Arithmetics came also many Newtoniana. One item of interest was NEWTON's copy of a Journal with his signature and numerous notes.

Through the courtesy of Dr. A. KOEGH, Librarian of Yale University, were loaned two remarkable books; copies of the *Principia*, 1713 edition, and of the *Opticks*, 1706 edition which

had been presented to the colonial college in New Haven in 1714 by Sir ISAAC himself.

Through the courtesy of Dr. F. C. NEWELL of Boston University, Dr. S. BRODETSKY, Leeds University, and of the Grantham Public Library, came an interesting collection of prints, pamphlets and newspapers giving an elaborate account of the NEWTON birthplace, of his boyhood and of the bicentenary celebration at Grantham on March 20, 1927.

Six interesting photostat copies of various leaves of a precious common-place note-book of NEWTON when a boy of 17 years, were exhibited. The original note-book is in the PIERPONT MORGAN Library, New York.

Mr. JAMES STOKELY of Science Service exhibited a copy of RICCIOLI's *Almagestum Novum* which came from NEWTON's Library with his own annotations and Dr. FLORIAN CAJORI, University of California, exhibited some items bearing upon NEWTON's religious writings.

Library of Congress
Washington, D. C.

FREDERICK E. BRASCH
Secretary.

Proceedings of the Council Meeting of the History of Science Society

The Council of The History of Science Society held its annual meeting on Saturday February 25, 1928 at 2.00 o'clock P. M. in the office of the President Dr. DAVID EUGENE SMITH, Teachers College, Columbia University, New York. The following members of the Council were present, Dr. DAVID EUGENE SMITH, Dr. EDGAR F. SMITH, Dr. LYNN THORNDIKE, Dr. GEORGE SARTON, Dr. J. MCKEEN CATTELL, Dr. ISAIAH BOWMAN, Dr. HARRIET BIGELOW, Dr. L. C. NEWELL, Dr. LAO G. SIMONS, Dr. J. K. WRIGHT and Mr. F. E. BRASCH.

At the request of the President the reading of the minutes of the last meeting of the Council held in Philadelphia, December 28, 1927, was dispensed with. The election of officers for the year 1928 followed with the counting of the ballots, namely for President, Dr. EDGAR F. SMITH, University of Pennsylvania, Philadelphia, Penn.; Vice-Presidents, Dr. JOHN C. MERRIAM, Carnegie Institution of Washington, Washington, D. C., and Dr. JAMES HARVEY ROBINSON, New York City, N. Y., also five new members for the Council to serve until 1930, Dr. J. MCKEEN CATTELL, New York, N. Y., Dr. G. S. BRETT, Toronto, Can., Dr. FLORIAN CAJORI, Berkeley, Calif., Dr. LAO G. SIMONS, New York, N. Y. and Dr. C. A. BROWNE, Washington, D. C. The Council in turn elected Dr. HARRY ELMER BARNES, Northampton, Mass., Recording Secretary, and Mr. F. E. BRASCH, Washington, D. C., Corresponding Secretary and Treasurer, each to serve for one year.

Following the election of officers, the various items of business prepared upon a memorandum sent in advance to each of the members of the Council were taken up and discussed. 1) Report of the Secretary and Treasurer. 2) Endowment plans, Life members and Honorary members. Committees were appointed to report at the next meeting. 3) Delegates to the International Congress

of Historical Sciences. This question was left for the President to consider. 4) Report on the NEWTON Memorial volume. 5) Endorsement and approval of the Society of Dr. G. WALCOTT's plans for the publication of Source Books in the History of Science. 6) Report of the plans for the next program of the Society which meets in New York in December 1928. It was recommended that a series of addresses be prepared on the History of Medicine.

FREDERICK E. BRASCH.
Corresponding Secretary.

An Arabic Compendium of Medico-philosophical Definitions.

Since the appearance of the biographical work of ḤUNAIN b. IṢḤÂQ, of which I recently gave an exposition and commentary (1), there has been published another small treatise from the hand of a Christian-Arabic physician. But this time it is one of the last scholars of the period of the transition of the Greek to the Arabic sciences (2) who takes such pains to explain the Greek thought to his countrymen.

The author is ' UBAID-ALLÂH IBN JIBRÂ'ÎL IBN BAKHTÎSHÛ' (3), the last descendant of the well-known family of the BAKHTÎSHÛ who furnished the Abbasid khalifs with court-physicians during the course of three centuries. His work is *Ar-rauḍa at-ṭibbiyya* (« The Medical Garden ») containing fifty definitions of philosophical and medical subjects. The editor of this little treatise, of which hitherto several MSS. were known in the Escorial, Gotha, British Museum and Paris National Library, is the Syrian priest Father PAUL SBATH who possesses a large collection of Arabic manuscripts. He recently published some remarks on the most important specimens in his collection (4). He prepared the edition from three of his own MSS., a little carelessly, in

(1) See *Isis*, 8, p. 685-724, 1926.

(2) *Ar-Raoudat at-tibbiyya* (*Le jardin médical*) par UBAID-ALLAH BEN GIBRAIL BEN BAKHTICHOU, Chrétien décédé en 1058. Texte arabe publié pour la première fois d'après trois manuscrits conservés dans la Bibliothèque des Manuscrits du Père PAUL SBATH, avec une introduction et des notes et index par le P. PAUL SBATH. Cairo 1927. H. Friedrich & Co. Libraires-Editeurs, 8° 73 p. (in Arabic).

(3) BAKHTÎSHÛ' or in Syriac pronunciation BOKHTÎSHÔ' means « Jesus Has Given ». The history of the family from the pen of C. BROCKELMANN is to be found in *Encyclopedia of Islâm* vol. I, 1913, p. 601-2.

(4) 1500 manuscrits scientifiques et littéraires, très anciens, en Arabe et en Syriaque, découverts par le R.P. SBATH. *Bull. de l'Institut d'Egypte* 1926, t. VIII. Session 1925-6, p. 21-43. Analysis by M. MEYERHOF *in* MGMN vol. XXV, 1926, p. 311

Oriental manner, without giving the variations, and sometimes correcting the original text in a not altogether judicious manner. But as he is perfectly acquainted with the classical Arabic language, he has produced a rather good and profitable text.

Before giving the gist of this MS., I will make some mention of the author's life which has been recorded at some length by LECLERC (5) and in brief by BROCKELMANN (see note 3).

I prefer to give an abridged translation of the available chronicles.

IBN ABÎ UṢAIBI'Ã (6), after having dealt with the long series of descendants of the BAKHTÎSHÛ family and particularly with the father of our author, JIBRÂ'ÎL B. 'UBAID-ALLÂH (d. 1005 A.D.), who was a distinguished practitioner in Baghdâd at the end of the Xth century A.D., speaks of 'UBAID-ALLÂH as follows :

« His full name is ABÛ SA'ÎD 'UBAID-ALLÂH B. JIBRÂ'ÎL B. 'UBAID-ALLÂH B. BAKHTÎSHÛ' B. JIBRÂ'ÎL B. BAKHTÎSHÛ' B. JÛRJIS B. JIBRÂ'ÎL (7) ; he was excellent in the medical profession, renowned for his prominent activity in it, perfect in its principles and branches, distinguished above all the nobles attached to this profession. He was, moreover, strong in the knowledge of the science of the Christians and their sects. He was of superior skill in the medical art on which he composed many books. He lived at Mayyâfâriqîn (8).

He was a contemporary of IBN BUṬLÂN (9) with whom he had intercourse and was familiar ; there existed an intimate friendship between them. 'UBAID-ALLÂH B. JIBRÂ'ÎL died in the course of the

(5) L. LECLERC, Histoire de la médecine arabe, T. I. Paris 1876, p. 372-4.

(6) *'Uyûn al-anbâ' fî ṭabaqat al-aṭibbâ'* (Sources of Instruction about the Classes of Medical Men). Ed. Cairo 1299-1882, vol. I, p. 148.

(7) This long name includes the pedigree of the author. His ancestor JÛRJIS B. JIBRÂ' ÎL B. BAKHTÎSHÛ' was chief physician of the hospital and medical school at Gondêshâpûr (Khûzistân in Persia), when he was appointed by the Caliph AL-MANSRÛR to his personal service in Baghdâd in 148 A.H. (765 A.D.). IBN ABÎ UṢAIBI'A attributes to him the first Arabic versions of Greek medical books. After his death (771 A.D.) his son BAKHTÎSHÛ' B. JÛRJIS was physician in ordinary to HÂRÛN AR-RASHÎD, and was followed by his son JIBRÂ' ÎL B. BAKHTÎSHÛ' (d. 830 A.H.) who was the most renowned member of this noble family.

(8) Town in Upper Mesopotamia, near Amida or Diyâr Bakr.

(9) ABÛ'L-ḤASAN AL-MUKHTÂR... IBN BUṬLÂN (d. about 1065 A.D.) was a celebrated Christian physician in Baghdâd, known by his *taqwîm aṣ-ṣiḥḥa* (« Almanach of Health »), the Latin version of which was published in 1531 at Strassburg under the title : *Tacuini sanitatis Elluchasem Elimithar* etc. See *Encycl. of Islâm*, vol. II, Leyden and London, 1916.

year four hundred and fifty and more (10). His works are the following : *A Treatise on the Difference between the Kinds of Milk*, which he composed for some of his good friends in 447 (A.H., i.e. about 1055 A. D.) ; *The Merits of Physicians*, in which he recorded some of their circumstances and memorable acts (10[a]). He composed this book in 423 (about 1031 A.D.). *The Garden of Medicine* which he wrote for the professor ABÛ' L-ḤASAN MUḤAMMAD B. 'ALÎ (11); *On the Right Way to Preserve Descent*, which he wrote in 441 (about 1049 A.D.) ; *Missive to the Professor Abû Ṭâhir b. 'Abd al-Bâqî known as Ibn Qaṭramîz* (12), in answer to his question ; *Missive explaining the Necessity of Respiratory Motion*; *Curiosities of Extemporaneous Questions out of the Scientific Principles of Medicine* ; *Memorial of the Resident and Provision of the Traveller* ; *Special Book on the Science of Properties* ; *On the Nature, the Properties and the Utility of the Organs of Animals*, composed for the Prince NAṢÎR AD-DAULA (13). »

BROCKELMANN (14) adds a treatise *On Love as a Disease* extant in a MS. in the Leiden University Library. He tells us that it appears that out of all his family some of the literary works of 'UBAIDALLÂH alone have survived for us, whilst the numerous works of his ancestors are lost (15). IBN AL-QIFṬÎ (16) records that 'UBAID-

(10) i.e. after the year 1058 A.D.

(10[a]) Often quoted by Ibn Abî Uṣaibi'a.

(11) This may be the Mohammedan grammarian ABÛ L-ḤASAN MUḤ. B. 'ALÎ AD-DAQÎQÎ mentioned by HADJI KHALFA (Lexicon Bibliographicum ed. FLUEGEL, T.V., p. 493 and 532. London 1850). He was born in 384 A.H., i.e. about 994 A.D.

(12) I was not able to find any record in Arabic biographical literature concerning this scholar whose name IBN QAṬRAMÎZ seems to be a nickname (« Son of the Tureen »).

(13) ḤASAN B. ḤUSAIN with the honorary title NAṢÎR AD-DAULA (« Defender of the State »), a descendant of the Hamdânid princes of Môṣul, was governor of Damascus (in 1042 A.D.) and later on (after 1062 A.D.) vizier and commander of the Turkish troops to the Fatimid Caliph AL-MUSTANṢIR in Cairo, where NAṢÎR was assassinated in 1074 A.D.

(14) *Encyclopedia of Islâm* (vol. I, p. 602), where he gives, as mentioned before, a complete bibliography of the BAKHTÎSHÛ' family.

(15) I found in a Cairo private library and in the Leningrad Academical Library (see IGN. KRAČKOVSKIJ, Les *manuscrits arabes de la collection de Grégoire IV, patriarche d'Antioche*. Leningrad 1924, p. 19) a MS. on Arabic ophthalmology from the hand of a certain 'ALÎ B. IBRÂHÎM B. BAKHTÎSHÛ' of Kafr-Ṭâb in Syria. But I have not yet been able to establish the relationship between this medical writer and the great Mesopotamian family of physicians.

(16) *Ta'rîkh al-Ḥukamâ* (History of Philosophers) ed. JUL. LIPPERT, Leipzig 1903, pp. 126-7, 356.

ALLÂH B. JÎBRÂ'ÎL had been asked by a man about GALEN and his period and that he answered by a *Missive* based on the historical work of a certain monk HÂRÛN IBN 'AZZÛN (17). It is very probable that this *Missive* is the one mentioned by IBN ABÎ UṢAIBI'A as being an answer to IBN QAṬRAMÎZ. The other Arabic bibliographies mention 'UBAID-ALLÂH only in a cursory manner. His most important work was without doubt the *Memorial of the Resident and Provision of the Traveller* (*Tadhkirat al-Ḥâḍir wa-Zâd al-Musâfir*) of which our *Medical Garden* is an extract.

We shall now analyse the contents of this little book which has just been edited for the first time. I have appended the Arabic and added the Greek technical terms to the title of each chapter, since they are missing from most Arabic dictionaries.

AR-RAUḌA AṬ-ṬIBBIYYA.

The editor P. PAUL SBATH says in a short introduction that he published the work in accordance with his own three MSS. dating from the XIIIth to the XVIIth century A.D. The oldest MS. comprises 41 pages with 19 lines each.

The author 'UBAID-ALLÂH B. JIBRÂ'ÎL B.BAKHTÎSHÛ' begins his book with the remark that he wrote it at the request of his professor (mentioned in note 11, p. 342) with the intention of giving a comprehensible extract of his own great work containing the principles of the medical art. He points out that he composed it in 50 separate chapters, each of them containing the explanation of a philosophical term in order to facilitate the learning of their meaning.

The chapters are as follows:

1. On the Kind (*jins*, γένος);
2. On the Species (*nau'*, εἶδος);
3. On the Difference (*faṣl*, διαφορά) ;
4. On the Property (*khâṣṣa*, ἰδιότης) ;
5. On the Accident (*'araḍ*, συμβεβηκός) ;
6. On the Substance (*jauhar*, οὐσία) ;
7. On the Quantity (*kamm*, ποσότης) ;
8. On the Quality (*kaif*, πόιοτης) ;
9 On the Nature (*ṭabî'a*, φύσις) ;

(17) IBN ABÎ UṢAIBI'A calls him HÂRÛN B. 'AZZÛR (vol. I, p. 72). It is possible that he is identical with the medical writer AHRÔN B. A'ÎN (« the Priest ») who lived in Alexandria at the beginning of the VIIth century, or with the Nestorian historian AHRÔN mentioned by BAUMSTARK (*Geschichte der syrischen Literatur*. Bonn, 1922, p. 241).

10. On the Body (*'uththa*, σῶμα) ;
11. On the Element (*istiqs*, στοιχεῖον) ;
12. On the Complexion (*mizâj*, κρᾶσις) ; warmth, cold moisture, dryness and their combinations ;
13. On the Mixture, Humour (*khilṭ*, χυμός) ; blood, phlegm, yellow bile and black bile ;
14. On the Organ (*'aḍw*, ὄργανον, μόριον); there are simple and compound organs Of the latter some are principal organs such as the brain, heart, liver ; and others are subordinate to them such as nerves, arteries and veins. The simple organs are bones, cartilages, fat, flesh, etc.
15. On the Power, Faculty (*quwwa*, δύναμις) ; its substance is imperceptible and it can be perceived only by its action or function. There are three kinds; the natural, the animal and the psychical faculties. The subdivisions follow GALEN's corresponding ideas.
16. On the Activity, Function (*fi'l*, ἐνέργεια) ; some of the functions are general and partly made up of several faculties, as for instance nutrition which is achieved by attracting and repelling faculties ; some are special to certain organs, such as vision, hearing, taste, digestion and blood-production, which last is peculiar to the liver.
17. The Spirit (*rûḥ*, πνεῦμα) is a thin, airy body circulating in the organs, which specifies the faculties for their function or activity. There are three spirits, the natural, the animal and the psychical. Then follows the explanation of their nature according to GALEN.
18. On the Soul (*nafs*, ψυχή).

I translate the first part of this chapter in order to show its resemblance to the corresponding chapter in PLUTARCH's *De Placitis Philosophorum* (18).

« The soul is according to the school of ARISTOTLE the production (*namâ*) of a natural body endowed with vital energy (19). This is the definition in the didactic sense ; its definition in the physical sense is that it is the origin of all sensation and motion. According to PLATO's school the soul is a simple intellectual substance which possesses its own motion following a harmonious numeric proportion (20). According to the school of PYTHAGORAS it is a luminous sub-

(18) PLUTARCHI Chaeron. Moralia ed. G. N. BERNARDAKIS. vol. V. Lipsiae MDCCCXIII, p. 332.

(19) *Ἀριστοτέλης ἐντελέχειαν πρώτην σώματος φυσικοῦ, ὀργανικοῦ, δυνάμει ζωὴν ἔχοντος*. Soul is the realization of the body (*ἐνέργεια σώματος*). See Aristotle's Psychology, ed. EDWIN WALLACE, Cambridge, 1882, p. XLI of the introduction.

(20) PLUTARCH *loco cit.* : *Πλάτων οὐσίαν νοητὴν ἐξ ἑαυτῆς κινητὴν, κατ' ἀριθμὸν ἐναρμόνιον κινουμένην*

stance (21). According to the school of THALES it is a physical substance in perpetual motion (22). According to the school of DICEARCHUS it originated from the four elements (23). Very many of the Ancients follow this school, as GALEN points out in his book « *That the Soul follows the Complexion of the Body* ». According to the school of ANAXAGORAS the soul is a hot spirit (*rûḥ ḥârra*). (24). The school of HERACLITUS, prominent in the medical profession, is of the opinion that it springs from the vapours of the humours ». (25).

'UBAID-ALLÂH concludes that the soul is a substance, since it is the bearer of qualities and accidentia and is self-sustaining.

19. On the Reason (*'aql*, *νοῦς*) ; definition according to ARISTOTLE and PYTHAGORAS.

20. On Appetite and Lust (*shahwa wa-ladhdha*, *ὄρεξις καὶ ἡδονή*).

21. On Love (*'ishq*, *ἔρως*). This chapter begins as follows : « On this *disease* (26) the Ancients are of different opinions, so that some of them say : it is proper to the rational soul (*nafs nâṭiqa*, *ψυχὴ λογιστική*), being a corruption of the imagination and thought. Others say : it is proper to the bestial soul (*nafs bahaimîyya*, *ψυχὴ θηριώδης*) (27), being a disease which befalls it from excess of lust. » The author then cites THEMISTIUS whom he identifies by a strange error with JOHN THE GRAMMARIAN or PHILOPONUS (28). After him the author

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(21) This is an error of 'UBAID-ALLÂH or of his sources. It is DEMOCRITUS who calls the soul a fiery compound (*πυρώδες σύγκριμα*).

(22) The name of THALES is mutilated in the Arabic text (*Sâwûs*). PLUTARCH *l.c.* : *Θαλῆς ἀπεφήνατο πρῶτος τὴν ψυχὴν φύσιν ἀεικίνητον...*

(23) This name is equally mutilated (*Dankârkhûs*). PLUTARCH *l.c.* : *Δικαίαρχος ἁρμονίαν τῶν τεσσάρων στοιχείων.*

(24) Here again 'UBAID-ALLÂH is not in accordance with PLUTARCH who says (l.c., p. 333) : *Οἱ Στωϊκοὶ πνεῦμα θερμόν.* ANAXAGORAS derives the soul from the *νοῦς* and calls it according to PLUTARCH « air-like » (*ἀεροειδής*).

(25) Originally HERACLITUS derives the soul from the eternal fire (*πῦρ ἀείζωον*), but occasionally he pretends that it springs from watery exhalations of the body (Fragm. XII : *καὶ ψυχαὶ δὲ ἀπὸ τῶν ὑγρῶν ἀναθυμιῶνται*). Many parts of the Corpus Hippocraticum reveal the influence of the doctrines of HERACLITUS in a marked degree.

(26) *'ishq* is the violently passionate love (*ἔρως*), *ḥubb* or *maḥabba* the affection (*ἀγάπη*, *φιλία*).

(27) I was not able to find this expression in the Greek philosophers. It is not rare in Arabic philosophical works where animal (*ḥayawânî*) is always distinguished from bestial (*bahaimî*), in speaking of the soul or the spirit. The low animal nature is mentioned as *τὸ θηριῶδες* in PLATO's *Cratylus* and ARISTOTLE's *Ethics*.

(28) This error even passed into IBN AL-QIFṬÎ's *Ta'rîkh al-Ḥukamâ* where we read (p. 356) : « 'UBAID -ALLÂH B. JIBRÂ' ÎL B. BAKHTÎSHÛ 'says that the name of YAḤYÂ (JOHN) is THEMISTIUS ».

quotes « MASAQSÂR THE INDIAN » (29) whose theory of love is an astral one. There follows ARISTOTLE's doctrine and the explanation of both of the afore-mentioned theories of love. The last paragraph of this chapter brings with it an interesting quotation from the *Kâfî* (« The Sufficient »), a great treatise on medicine in five volumes composed by 'UBAID-ALLÂH's father JIBRÂ'ÎL (30) ; this learned medical man points out that love is a corruption caused by the influence of the senses, especially the sight, on the rational soul which is subdued to the morbid passion « like a great king when he becomes an instrument in the hands of a vile slave. »

22. On Sensation (*ḥiss*, αἴσθησις) ;
23. On Imagination (*takhayyul*, φαντασία) ;
24. On Thought (*fikr*, νόησις) ;
25. On Memory (*dhikr*, μνήμη) ;
26. On Character (*khulq*, ἦθος) ;
27. On Anger (*ghaḍab* ὀργή) ; this is of two kinds which are opposite, scorn (*anafa*, θυμός, πικρία) ; and wrath (*tahawwur*, ὀξυθυμία).
28. On Motion (*ḥaraka*, κίνησις).
29. On Time (*zamân*, χρόνος) ; it is to be distinguished from the exact or critical time (*waqt*, καιρός) in accordance with Aristotle's distinction.
30. On Space (*makân*, χώρα) ;
31. On Genesis and Decay (*kaun wa-fasâd*, γένεσις καὶ φθορά) ;
32. On Science (*'ilm*, ἐπιστήμη) ;
33. On Knowledge (*ma'rifa*, γνῶσις) ;
34. On Conjecture (*ḥads*, ὑπόληψις) ; the most important by induction (*istidlâl*, ἐπαγωγή, ἐπακτικώς) ;
35. On Syllogism (*qiyâs*, συλλογισμός) ;
36. On Argument (*burhân*, ἀπόδειξις) ;
37. On the First Cause (*'illa*, ἀρχή) ;
38. On Sleep (*naum*, ὕπνος) ;
39. On Vision (*ru'yâ'*, ὄνειρος, ὅρασις) ; this is the longest chapter dealing with the divine vision, the natural vision or dream, and the corporal vision viz., nightmares and hallucinations and their causes.
40. On Pulse (*nabḍ*, σφυγμός) ;
41. On Crisis (*buḥrân*, κρίσις) ;
42. On Disease (*maraḍ*, νόσημα) ;

(29) This name is to be found nowhere in the Arabic medical or biographical literature. I presume it to be a mutilation of an Arabic transcription (*Qâmashtâr* or the like) of *Kâmashâstra*, the Indian science of love, or the famous *Kâmasûtra*, the guide of love.

(30) Mentioned by IBN ABÎ UṢAIBI'A, vol. I, p. 147.

43. On Cause (*sabab*, αἰτία) ;
44. On Symptom (*'alâma*, σύμπτωμα) ;
45. On Health (*siḥḥa*, ὑγεία) ;
46. On Nutriment (*ghidhâ'*, τροφή) ;
47. On Remedy (*dawâ*, φάρμακον) ;
48. On Nutrition (*al-ghâdhî w'al-mughtadhî*, lit. « the nourishing and the nourished », θρέψις) ;
49. On Sight (*ibṣâr*, ὄψις) ;

Of this chapter I give a full translation, as it is characteristic of the manner in which the author represents Greek thought throughout his compendium.

« Sight is the accession of the seen objects to the sense of vision (*ḥâssat al-baṣar*). This is due to a ray which springs from the eye and falls on the visual objets (*mubṣarât*, βλεπόμενα).

A person sees only that on which the visual ray (*shu'â' baṣarî*, ἀκτίς) falls, but he does not see that on which it does not fall. Therefore he is unable to see the objects between which and the sight exist separations preventing the visual ray from falling on them. The opinions of the Ancients on the nature of sight differ, and everyone of them believed firmly in some theory. Among them are DEMOCRITUS and EPICURUS whose theory is that the visual faculty (*quwwa baṣariyya*, ὁρατικόν) exists by means of images which are shaped in the visual ray and return to the visual organ (*baṣar*, ὄψις). EMPEDOCLES' (31) opinion is that the visual ray is blended with the patterns which are shaped in it, and he therefore calls this connecting ray the image-ray (*shu'â' dhû tamâthîl*) (32). HIPPARCHUS thinks that the visual rays leave both eyes, and stretch themselves until they meet the visual objects with their extremities, and that they are like hands which feel the things outside the body and transmit this (the feeling) to the visual faculty. (33). GALEN confirms the exactitude of this theory by mathematical arguments in his book *On the Utility of the Parts of the Body* (34). PLATO expresses the opinion that vision is effected by an association of the visual light (*ḍau' baṣarî*) with the aerial light (*ḍau' hawâ'î*) and its emanation into it, by reason of the affinity existing between them ; and that the light which is

(31) The name is mutilated to Dâqlûs and could be read Diocles.

(32) PLUTARCH *loc. cit.*, p. 340 : *Ἐμπεδοκλῆς τοῖς εἰδώλοις τὰς ἀκτῖνας ἀνέμιξε, προσαγορεύσας τὸ γιγνόμενον ἀκτινείδωλον συνθέτως.*

(33) This is the so-called theory of the feeler which is sometimes attributed to EPICURUS. See HIRSCHBERG, *Geschichte der Augenheilkunde im Altertum*, Leipzig, 1899, S. 150.

(34) GALENI *De Usu Partium*, l. X, c. 12.

reflected from the objects stretches itself into the air by reason of its emanation, is rapidly transformed, and meets the fiery visual brightness (*ḍiyâ'*, *τὸ πυρῶδες τῆς ὄψεος*). This theory is called the Platonic meeting of the light-rays (35).

It is evident that 'UBAID-ALLÂH here follows PLUTARCH's De Placitis Philosophorum l. IV, c. 13 (Moralia vol. V). (36).

50. On the Voice (*ṣaut*, *φωνή*).

This is the aerial tone provoked by animals possessing lungs ; that which is provoked by inanimate things or animals possessing no lungs is called *qar'* (sound, *ψόφος*). Then follows an explanation of the cartilages which produce the voice, and of GALEN's opinion that a cartilage is more fit to provoke voice, because it is neither very hard nor very soft (37).

The book then ends with the words :

« Praise be to the Original Cause of the First Causes ! »

It would take too long to give a complete analysis of all the fifty chapters of 'UBAID-ALLÂH's guide to medico-philosophical definitions. It is obvious from the foregoing synopsis that he follows almost entirely the lines of Greek philosophy. PLATO is quoted six times, ARISTOTLE five times, PYTHAGORAS twice, the other philosophers are cited once only ; GALEN is quoted in five places, the author's father JIBRÂ'ÎL B. BAKHTÎSHÛ' once. But most of the chapters follow Aristotle's doctrines. It is not possible in this short summary to cite the works from which the quotations are taken. Most of them are second-hand quotations, showing that the author utilized Arabic translations of the philosophers and of their Greek commentators. None of his Arabic forerunners e.g. AL-KINDÎ (d. 873 A.D.), AL-FÂRÂBÎ (d. 951 A.D.) or the works of the *Brethren of Purity* (Xth century A.D.) are quoted by him. So we have to conclude that he used pre-eminently the translations, the diligent work of so many Nestorian scholars of the IXth and Xth centuries. The philosophical translators are principally ḤUNAIN B. ISḤÂQ and his pupils, particularly his son ISḤÂQ B. ḤUNAIN, ABÛ 'UTHMÂN SA'ÎD and 'ISÂ B. YAḤYÂ ; also QUSṬÂ B. LÛQÂ, ABÛ BISHR MATTÂ and his disciple YAḤYÂ B. 'ADÎ ; IBN AL-BAṬRÎQ, IBN NÂ'IMA, 'ISÂ B. ZAR'A, ABÛ'L-KHAIR

(35) The Greek expression is *συναύγεια*. (PLATO's *Timaeus*, XVI, 45).
(36) This treatise had been translated into Arabic by QUSṬÂ B. LUQÂ.
(37) See GALEN's *De Usu Partium* l. VII, c. 6.

B. AL-KHAMMÂR, AYYÛB AR-RAQQÎ, and others. They had created an almost complete canon of Greek philosophy in Arabic. This canon comprises the whole output of ARISTOTLE, of his commentators, (such as THEMISTIUS, ALEXANDER OF APHRODISIAS, PORPHYRY, and JOHN THE GRAMMARIAN), and many works of PYTHAGORAS, PLATO, DEMOCRITUS, THEOPHRASTUS and minor thinkers. 'UBAID-ALLÂH was probably unable to utilize the philosophical works of his contemporaries, the eminent Persians IBN SÎNÂ (38) and AL-BÎRÛNÎ, the Egyptian 'ALÎ B. RIḌWÂN and the world-famed mathematician IBN AL-HAITHAM of Baghdâd who lived in Egypt.

We regret that 'UBAID-ALLÂH's great treatise on ancient philosophy, « Memorial of the Resident and Provision of the Traveller » is lost. But even from the small extract «The Medical Garden » we see that the Greek philosophical tradition was alive fully in the middle of the XIth century, and that its theories were well understood and taught. Very soon the decay set in, and about the year 1100 A.D. orthodox scholasticism was already victorious in the Islamic world and led to the scientific stagnation which prevailed during the rest of the Middle Ages.

(*Cairo*) MAX MEYERHOF.

(38) AVICENNA himself wrote a compendium of purely philosophical definitions, the fourth of the *Nine Missives on Philosophy and Natural Science* of which many Arabic editions exist. (My own copy, *Ibn Sînâ, tis' rasâ'il*, is dated Cairo, 1326-1908). His definitions e.g. of the soul, mostly follow ARISTOTLE and are different from those given by 'UBAID-ALLÂH B. JIBRÂ'ÎL.

The "Alchemy" ascribed to Michael Scot.

MICHAEL SCOT, astrologer of the Emperor FREDERICK II and translator of ARISTOTLE, AVERROËS, and AVICENNA (1), appears in certain mediaeval manuscripts as the author of works on alchemy. In this there is no instrinsic improbability for a man of SCOT's occupation and surroundings, and a brief summary of alchemical doctrine actually occurs in a chapter of one of his authentic works on astrology (2), written between 1227 and 1235 and thus affording an early example of such material in Latin. Caution, however, is imposed upon us by the various false attributions which appear in SCOT's name, as well as by the confusion and uncertainty which still reign respecting the Latin literature of alchemy in general. We are still far from having attained a secure footing in this field. Meanwhile this paper proposes to describe more adequately a treatise, or pair of treatises, attributed to SCOT which are too long for publication here *in extenso* (3), and to call attention to the collaboration which they reveal between Italian alchemists and Jewish and Saracen experimenters.

The *Alchemy* ascribed to MICHAEL SCOT has reached us in two manuscripts. One is the well known fourteenth-century collection of alchemical works described by CARINI when it was in the possession of the Speciale family (4) and now MS Qq. A. 10 of

(1) J. WOOD BROWN, An Enquiry into the Life and Legend of MICHAEL SCOT (Edinburgh, 1897); HASKINS, MICHAEL SCOT and FREDERICK II, in *Isis*, 4, 250-275, revised in Studies in the History of Mediaeval Science (second edition, Cambridge 1927), ch. 13.

(2) Printed in *Isis*, 4, p. 271; Mediaeval Science, p. 295, cf. pp. 280, 281.

(3) For the *Liber luminis luminum* which SCOT is said to have translated, and a *Quèstio curiosa* which is clearly not his, see BROWN, *op. cit.* ch. 4, and his Appendix iii.

(4) I. CARINI, Sulle scienze occulte nel medio evo, Palermo, 1872; description repeated in G. DI MARZO, I MSS. della Biblioteca comunale di Palermo, 1878, iii, p. 220-243. For many kind offices in securing photographs I am indebted to my friend, Professor C. A. GARUFI. The treatise begins on f. 357.

the Biblioteca comunale at Palermo. This rich corpus of Latin alchemy, which also comprises (5) a catalogue of the alchemical library of seventy-two treatises belonging to a monk of S. Proculo at Bologna, is a mine of unexplored information ; its small format (133 × 94 mm.) indicates that it was meant as a pocket *vade-mecum* of the art. The other manuscript, of the early fifteenth century, is in MS. 125 of the library of Corpus Christi College, Oxford (6). Neither scribe appears to have understood all that he copied, and the text is in some places hopeless. I have based the following extracts primarily upon the Palermo codex (P) as generally more correct, giving the variants of the Oxford MS. (C) only where they help toward a better reading. Nevertheless, though offering a poorer text, C appears to stand in one respect nearer the original in that MICHAEL SCOT there speaks in the first person, while in P he is cited in the third. The text begins :

Incipit liber magistri MICCAELLIS SCOTTI *in quo continetur magisterium. Incipit liber magistri* MICCAELLIS SCOTTI *de arte alchimie secundum quem in diversis provinciis cum phylosophis huius artis est operatus. Incipit liber magistri* MICCAELLIS SCOTTI. (7). (I) Dum animadverterem nobilem scientiam apud Latinos penitus denegatam, vidi neminem ad perfectionem posse pervenire propter nimiam confusionem que in libris philosophorum reperitur, extimavi secreta nature intelligentibus revelare incipiens a maiori magisterio (8) et minori que inveni de transformatione metallorum et de permutatione ipsorum qualiter substantia unius in alteram permutetur. Hoc enim nullis vel paucis erat cognitum. Reperitur autem in libris philosophorum de permutatione et transformatione metallorum, sed (9) in eorum philosophia tanta erat obscuritas et oculi hominum caligine obfuscati et corda eorum velamento ignorantie oppilata quod ars alkimie(10) nullis vel paucis posset(10a) revelari. Multi erant operantes ignorantia et obmittebant in operibus eorum et tempus eorum preteribat in obmissione (11) operum. Non potest aliquis sine magistro esse peritus, maxime is qui ignorat illud quod facit, si non est expertus, obmittit in eo. Cum prius enim animadverterem et perquirerem in libris philosophorum ut predictum, volui ipsam obscuritatem meo animo declarare gradiens et perquirens undique et ultramarinis partibus cum viris philosophis et sapientibus latinis iberiis (11a) harabicis saracenis

(5) F. 370 v. – (6) F. 97-100 v. — (7) Titles and chapter-headings only in P. The numbering of the chapters is mine ; — (8) P, ministerio ; — (9) Om. P ; — (10) hominum... alkimie, om. P ; — (10a) P, poterant ; — (11) P, admissione. C, amissione ; — (11a) P, iberniis. C, om. ; —

armenicis theophilis grecis et undique partibus provinciis et linguis, hiis omnibus perquisitis eorum prophetiam (12) meo corde notavi.

(II) *Item prologus in quo demonstratur secretum philosophorum.* Creator omnium Deus, qui ex nichilo(13) nova condidit universa, ante(14) ipsarum generationem de rerum (15) statu iudicans hoc quidem de universitatis sue tesauro largiri(15a) dignatur et singulis distribuit unde omnis creatura eidem exibet obedientiam, ymaginavit(16) priusquam fierent cunta habens eorum notitiam arcano cordis qui suum spiritum cum intellectione (17) infundit habite tandem creature. Hic motus existit ut summentes et venientes (18) scriptorum instructiones huius compositionis industriam quasi quadam compagine sociaret,ut ablata totius alterationis rixa rationale animal positivamque iusticiam (18a) nexu equali adinvicem federaret, universos itaque stolidos tamquam sapientes ad probandos facere contigisset quod nos eruditorum prudentium secreta computanda alkimie artem rationandi secreta nature (19) mentis arcano revocans, loca fixa (20) directos gradus ortus occasus permutationes et etiam distillationes et que sunt in eis alterationes admiranda vestigia attendens alchimie statum minus prudentium (20a) deprehendendi (21) errores. Hac igitur permutatione ratione cogente compendium hoc certissimum ex hiis omnibus prudens invenit antiquitas. Deinde aput omnes filosoficas permutationes ratum arbitror (22) quicquid in hac arte conditum subsistendi vicem alkemiarum (22a) vel alkemistarum. Est autem difficile exemplar (23) in libris antiquis philosophorum contineri (24) quia (25) artificium alkimie antiquius forte antiquitas refert. Ego vero (26) magister MICCAEL SCOTTUS interpretationem aggredior et tibi magistro TEOPHILO gayto Saracenorum tecum Tunixe (27) huius munusculum apporto et secreta nature et verba philosofica que audivi tecum volo alchimiam translatare. Hec est solutio caliditatis et roritatis et balneum aquosum et locus roridus et humidus et vaporosus. Hic est puteus solutionis et fimi acervus, et hic est fons in quo latet anguis cuius venenum omnia corpora interficit, et hoc est secretum omnium secretorum huius scientie, et hec est res quam in libris suis semper occultaverunt phylosophi ne facile possit ab aliquo tantum secretum haberi. Huius namque (28) tesaurus rei et in hac arte et in re est omne secretum,et hec est res que erigit de stercore pauperem(29) et ipsum regibus equiparat et hec est res per quam pater THOLOMEUS et

(12) C, philosophiam ; — (13) P, ex°. C, cunta ex nichilo ; — (14) P, qñ ; — (15) P, illarium; — (15a) P, largitu; — (16) P, ymaginavi; — (17) P, quod qui suspectum eum cum intelliget terre; — (18) ? C, ut sumat in hac ; — (18a) Om. P; — (19) P, vere. C, natura; — (20) PC, fixus; — (20a) P, prudentem ; — (21) P, deprehenditur ; — (22) P, inter arbitrorum ; — (22a) Om. P ; — (23) P, dissimile extis ; — (24) P, continere ; — (25) P, quia ergo. C, et quia ; — (26) Om. P; — (27) C, magno THEOPHILO regi Saracenorum de Tunucii ; — (28) C, Hoc enim est ; — (29) P, paupertatem ; —

HERMES dixerunt (30) se super omnes mundi circulos esse exaltatos (31). Te ergo, quicumque es, ad quem tantum secretum nature (32) pervenerit per fedus (33) Dei te rogo et coniuro ne ostendas hoc et si ostenderis non aperias cuiquam in aliquo vel nescio vel stulto aut avaro vel regi vel (34) potenti, ne socies tibi in hoc opere quempiam malum(35), prius referens gratias (36) Deo qui hoc te habere concessit operare, tum (37) secretissime perveniens ad illud quod amas et desideras cum auxilio Dei et potentia domini nostri IHESU CHRISTI qui vivit et regnat per omnia secula seculorum (38).

(III) *Quomodo metalla assimilantur planetis.* Septem sunt planete, scilicet (39) Saturnus Iupiter Mars Sol Venus Mercurius et Luna, Signa planetarum sunt duodecim, scilicet (39) Aries Taurus Gemini Cancer Leo Virgo Libra Scorpio Sagittarius Capricornus Aquarius Pisces.

(IV) *De naturis planetarum et metallorum.* A Sole incipiamus qui est nobilior et dignior omnium planetarum (40). Sol est calidus et siccus in quarto gradu, id est aurum (41). Luna est (42) frigida et humida in quarto gradu, id est argentum. Iupiter est (42) frigidus et humidus in tertio (43) gradu, id est stagnum. Venus est (42) frigida et humida in quarto gradu, id est es. Saturnus est (42) frigidus et siccus in quarto gradu (44). Mars est calidus et siccus in quarto gradu, id est ferrum. Mercurius est calidus et humidus in quarto gradu, id est argentum vivum (45). Omnia ista sunt firmata et alligata quatuor elementis et inde retinent suam naturam et proprietatem in calido et sicco in calido et humido in frigido et humido in frigido et sicco (46).

(V) *De maiori magisterio qualiter venus mutatur in solem.* Diximus superius de planetis et signis eorum cuius naturam et proprietatem habent, et in alio libro a nobis translato dissimus de naturis salium quomodo et qualiter in arte alkimie operantur maiori magisterio (47). Ad presens qualiter venus in solem mutatur et quomodo et qualiter fit artificialiter et que in hac arte sunt necessaria, tibi, frater Helya, diligenter et subtiliter enarravi. Accipe sanguinem hominis ruffi et sanguinem bubonis ruffi comburentem croceum vitriolum romanum colofoniam calcum bene pistatum allumen naturale vel(48) allumen romanum idem(49)

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(30) P, philosophorum Hermes dixit ; — (31) P, exaltatum ; — (32) P, tanti secreti naturam. C, tanta secreta nature pervenerint ; — (33) C, fidem ; — (34) Om. P. ; — (35) C, ne facias tibi in hoc opere copiam malorum ; — (36) P, gratiam in ; — (37) P, cum eo ; — (38) P om. domini... seculorum ; — (39) P, om. — (40) C, om. A... planetarum ; — (41) P, om. id est aurum, and other such ; — (42) P, om. ; — (43) C, quarto ; — (44) C, om. Saturnus... gradu ; — (45) P, om. Mars... vivum ; — (46) C, om. Omnia ... sicco ; — (47) C, Dicto de planetis de maiori magisterio et figuris, habita et notitia de salibus vel salium prout in aliquo libro a me translato dixi quomodo de salibus oportet in arte alkemie operari ; — (48) C, om. colofoniam... vel ; — (49) item ? ; —

allumen zuccherinum allumen de Castiglio tartarum rubeum markasidam (50) auream allumen de Tunixe quod est rubeum et salsum. Omnia ista pista simul in mortario eneo et subtiliter cribra cum panno subtili vel cum straminea (50a). Postea accipe pulverem istam et inpasta cum urina taxi vel cum succo cucumeris agrestis (51), et illa urina sit bene cocta cum sale et optime per filtrum distillata et sucus similiter sit distillatus et optime coctus, et cum pulvis inpastatus fuerit ad celestem ignem fac ipsum bene siccare vel ad vehentissimum calorem si non esset estas, et cum siccatus fuerit optime iterum tere ipsum et subtiliter cribra. Postea accipe de pulvere isto et mitte in crucibulo cum venere et statim suffla cum manticello per unam horam et cum liquefactum fuerit prohice superius de urina vel de suco cucumeris ut dictum (52) et iterum abde in fortem ignem per horam maximam prohiciendo in crucibulo de comburente satis (53) et de arsenico rubeo si haberes. Postea extrahe ipsum de crucibulo et si non erit bene coloratum funde iterum(54) cum tuchia et arsenico (55) rubeo et parum de predicto (56) pulvere et tribus salibus que operantur in sole donec erit bene coloratum, et iste sol postea poterit substinere omne iudicium. Si vis scire si est perfectus(57) pondera ipsum postea (58) funde ipsum fortiter ter vel quater et si tantum erit quantum erat in principio bonus et perfectus erit.

(VI) *De salibus*. Isti sunt sales qui in sole operantur : sal acrum, sal picrum de poncto (59), sal nitrum foliatum, sal alkali, sal rubrum (60), sal nacticum, sal alembrot vel de marrech (61). Ponas loco unius istorum salium de alumine rubro vel romano (61). Hoc est documentum magistri Miccaellis Scotti quod ipse super solem fecit et hoc documentum ipse docuit Fratrem Heliam et ego vidi Fratri Helie multis vicibus operari et hoc est experimentum a me probatum ; verax inveni. (62)

Then follows (c. VII) a closely similar *Minus magisterium* for turning mercury into silver, ending in the Oxford manuscript, *prout Michael predictus probavi et docui Frater Helya*, and in the Palermo text, *hoc est documentum magistri* Miccaelis Scotti *in mercurio*. The next chapter (VIII), on transforming copper into silver, is also parallel but bears no indication of authorship. Then comes a *congelatio* which begins (c. IX), *Hoc est documentum Baesis Saraceni de Maiorica*. Chapter X contains *Dealbatio eris perfecta secundum Barbanum Saracenum de Alap qui valde fuit sapiens et*

(50) P, martham; — (50a) P, staminia ; — (51) P, agresti; (52) P, om. ; — (53) C, pulverizato ; — (54) P, ipsum ; — (55) P, arte ; — (56) P, om. ; — (57) P, erit perfectum ; — (58) P, prius ; — (59) C, medium de puncto ; — (60) P, rubeo ; — (61) P, om. ; — (62) C, Et ego magister Michael Scotus sic operatus sum solem et docui te, Frater Elia, operari et tu mihi sepius retulisti te ins-

peritus in hac arte, a process for turning copper into silver which ends with this note ;

Nota quod dealbatio ista est perfecta et ego paucos inveni qui scirent ipsam facere sed ego vidi ipsam facere Fratri Helie et ego multotiens sum expertus et ipsam omnibus modis veracem inveni.

Chapter XI on making silver out of tin begins and ends as follows :

Hoc est documentum THEODORI Saraceni de Tunixe (63) qui valde sapiens et peritus fuit in hac arte, operatio stangni ad stridorem perfecte tollendum et ipsum perfecte dealbandum. Accipe sucum jusquiami sucum sorbarum sucum malorum granatorum...

Nota quod pulvis iste est tesaurus pretiosus in arte. Ipse laborat perfecte in sole et optime constringit mercurium et ultra modum dealbat erem et defendit ipsum a suis superfluitatibus. Apud Sarzanum vidi ipsum facere a quodam Iudeo qui vocabatur magister IACOBUS(64), et ipse me docuit, et ego multotiens sum expertus hoc experimentum et ipsum veracem inveni. (Injunction of secrecy).

Chapter XII describes the making of gold out of lead *secundum Modifar*, or (C) MEDIBABAZ, *Saracenum de Africa* (65). Here the attestation in the Palermo codex is in the name of Master G, but the Corpus manuscript has : *et ego* MICHAEL SCOTUS *multotiens sum expertus et semper veracem inveni.*

Chapters XIII and XIV deal with gums and *tuchie* respectively. Chapter XV on salts runs as follows :

(XV) *De salibus ad hoc magisterium.* Hec est affinatio salium qui in arte alchimie operantur. Accipe alba rotunda et in vase mundo mitte ubi sit aqua et postea pone ad ignem et fac subtus ignem donec sint dure et postea ipsas extrahe et optime munda ipsas album per se et rubeum per se et mitte album in petia subtili vel straminea et super turbidum calorem mitte et fac ita quod turbidus non ascendat et fac subtus postea bonum ignem. Accipe urinam tassi iuvenis et plenam manum(65a) salis communis prohice ibi intus et fac ipsum totum liquifieri. Cum liquefactum fuerit totum prohice super alba rotunda et illud quod cadet prohice post in ea (66). Postea stringe petiam et fac aquam exire (67) de alba substantia et aquam illam serva et cum aqua ista facta poteris affinare sales tuos

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tabiliter multis vicibus operasse ; — (63) C, THEODOSIUS Sarracenus de Cunusani ; — (64) C, et ego vidi istam operationem fieri apud Cartanam a magistro IACOBO Iudeo ; — (65) Translated by BROWN, p. 93 ; — (65a) P, plena manu ; — (66) P. om post in ea ; — (67) P, ex ere ; —

qui in luna operantur, et de rubea substantia idem facies et poteris cum ista alia aqua quam extracseris de rubea substantia affinare sales qui in sole operantur. *Explicit tractatus magistri* MICHAELIS SCOTI *de alk* (68).

Here the treatise ends in the Oxford manuscript, while the Palermo codex goes on in another hand with a similar treatise which it ascribes at the end to MICHAEL SCOT, who is also mentioned once in the course of the text (69). The heading of the first chapter tempts us to identify this with *alio libro a nobis translato... de naturis salium* cited above (c. V), but there is relatively little on salts, and the reference is more probably to the *Liber luminis luminum* (70). The second work, of approximately the same length as the first, begins as follows :

(I) *De salibus qui operantur ad solem.* Maxime in alkimia invenitur ad convertendum venerem in solem, mercurium in lunam (71), martem in lunam(71). In permutando ista tria non indiget sublimatione nisi (?) fictio operum. Multi autem solverunt et sublimaverunt et cum difficultate invenerunt, multi autem non solverunt et melius invenerunt. Potest autem quelibet substantia in alteram resolvi, in oleo, aqua, et sale, et per resolutiones. Sublimationes penitus non commendo sed commendo fictiones. Multa corpora vim amittunt (72) et suam naturam; probatum est; experto crede magistro. Possunt enim corpora resolvi in aqua et oleo. Omnes autem sublimationes expertus fui sed parum utilitatis inveni nisi in lapidibus faciendis et congelandis. Sublimationes que opportune sunt in arte tibi ad intelligentiam enarrabo.

(II) *Capitulum vitri.* Tere et ablue vitrum cum aceto sorbarum et mali granati et aceto rubeo octo vicibus et sicca ad solem. Per activitatem illorum acetorum subtiliantur et depurantur omnes superfluitates partium. Deinde funde in fortissimo crucibulo ferreo et extingue in aqua salis communis et m^a. et ar^a. vii vicibus. Iuro tibi quod in septem vicibus erit calcinatum in calce solis cui non est par per activitatem salium.

The titles of the succeeding chapters are :
(III) Capitulum vitri. (IV) Capitulum vitri quod operatur in solem. (V) Capitulum sublimationis mercurii. (VI) Capitulum sublimationis. (VII) Capitulum distillationis. (VIII) Preparatio vitrioli. (IX) Capitulum olei albi fixi philosofici. (X) De alembrottis. (XI) Capitulum de acetis. (XII) Capitulum lune. (XIII) Capitulum dulcificationis. (XIV) Modus purgandi mercurium. (XV) Capitulum sublimationis. (XVI) Capitulum

(68) Explicit only in C.; — (69) f. 360-363, where it ends: ... simili vase et dimittatur per totam. *Explicit liber magistri* MICCAELLIS SCOTTI ; — (70) Printed in SCOT's name in BROWN, appendix III ; — (71) MS. luna ; — (72) MS. admittunt.

lune tingende in aurum verum. (XVII) De modo pulverizandi aurum sive calcinandi. (XVIII) Capitulum grossi de croco ferri. (XIX) De modo margassite. (XX) Capitulum congelationis saturni. (XXI) Congelatio mercurii cum herbis.

HERMES is cited in chapters VIII and XII, 'BARRECTA Saracenus de Africa' in chapter III. In chapter VI we read :

Ego BALAC Saracenus de Regis Cibilia Gauco Pogis acc[epi ?] coagulationem tibi Fratri Elie transmisi et ipsam multotiens expertus fui, veracem inveni.

Chapter XX directs :

Pone in fornacem quam habuimus a magistro IOHANNE ALEXANDRINO designatam (?) que habet duos muros, unum de intus et alium de foris, sicut ego designavi discipulis magistri RATOALDI MEDIOLANENSIS.

In chapter IX occurs the only reference to MICHAEL SCOT :

Nota quod BARAC Saracenus et magister BOALA de Alap philosophi concordati sunt cum magistro MICCAELE SCOTTO quod terra que invenitur in allumine rubeo valde est bona mutando plumbum in optimum solem et lunam, albedinem perfectam dat eri, et optime constringit mercurium.

Chapter XVIII contains what may be a reference to SCOT's studies at Toledo :

Illud estanum postea vidi Tollecti et contulit ista micchi et eadem contulit cuidam consanguineo suo seni et ille senex cum eodem crocio operati sunt Tollecti (73) secundum modum predictum.

What we have here is not a comprehensive or systematic treatise like those ascribed to « GEBER » (74) or the briefer one of Master SIMON OF COLOGNE (75), nor yet an orderly description of salts and alums like the *Liber luminis luminum* (76), but rather accounts of particular experiments and processes such as BERTHELOT has

(73) MS. Celletti. No subject for contulit appears in what precedes.

(74) E. DARMSTÄDTER, Die Alchemie des Geber (Berlin, 1922 ; Isis, 5, 451-455); id., « Liber Misericordiae GEBER », in *Archiv zur Geschichte der Medizin*, 18, pp. 181-197 (1925).

(75) ED. SUDHOFF, in *Archiv für die Geschichte der Naturwissenschaften*, 9, pp. 54-67 (1922).

(76) BROWN, app. III.

indicated under the name of various Italians of the thirteenth century (77). Any special study of these processes must be left to those familiar with other contemporary treatises. In general, the materials and methods are reasonably clear. Besides the metals themselves, the authors use earths, alums, glass, fruit acids and vegetable juices, gums, and *tuchie*. They are acquainted with solution, fusion, filtration, sublimation, distillation with the alembic, and calcination. The first treatise makes no use of weights and measures, the second frequently mentions specific quantities, usually pounds.

Besides illustrating the processes of alchemy in the thirteenth century, these two treatises may throw some light on its sources. They give us neither the translation of formal Arabic works nor the independent experiments of Latins working by themselves in the West. On the contrary the Latins are apparently in close contact with Jewish and Saracen experimenters. They watch MASTER JACOB the Jew at Sarzana ; they cite specific experiments, or *documenta*, of Saracens of Africa, Tunis, Majorca, and Aleppo ; they note the agreement of BARAC and BOALA with MICHAEL SCOT. SCOT has been at Toledo, and if we can trust the preface, he dedicates his *Alchemy* to a Saracen official of Tunis, and has been in contact with alchemists of other lands. All this points to an amount of coöperation and interchange which has not heretofore been noted in the field of alchemy but which can easily be paralleled in other sciences in the same period. One need only recall the Jewish and Mohammedan scholars at the court of FREDERICK II and that emperor's correspondence and questionnaires addressed to the courts of Mohammedan rulers, including Tunis. (78)

With this milieu and these connections the attribution to FREDERICK's astrologer MICHAEL SCOT offers no inconsistency. Furthermore, we know that SCOT was familiar with the principles and practices of alchemy ; that he had an experimental habit of mind ; and that the form « ego MICHAEL SCOTTUS » appears in his authentic writings (79). Comparison with his other works is inconclusive, since the *Alchemy* is unsystematic in form and has

(77) La chimie au moyen âge, I, pp. 75-78.
(78) Mediaeval Science, chs. 12, 14, especially p. 252-254, 290.
(79) Mediaeval Science, p. 272.

in any case been reshaped by another hand. Furthermore, many false ascriptions gathered about SCOT's name and reputation as a wizard, and his contemporary Friar ELIAS OF CORTONA, who also appears in the experiments noted above, became the centre of a suspicious alchemical literature (80) ; so that it may be well to suspend our judgment as to the author until the discovery of further evidence. More important than the matter of individual authorship are the indications of collaboration with Jewish and Saracen experimenters in the West.

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(80) LEMPP, Frère ELIE DE CORTONE, Paris, 1901 ; GOLUBOVICH, Biblioteca bio-bibliografica della Terra Santa, Quaracchi, 1906, I, p. 116-117, 223-224 ; and for relations with FREDERICK II, A. HASELOFF, Die Bauten der Hohenstaufen in Unteritalien, Leipzig, 1920, I, pp. 34-37. Of the alchemical works ascribed to Friar ELIAS two are in Ms. 104 of the University of Bologna, f. 138 v, « Liber Fratris HELYE de Asisio Ordinis P. Minorum de secretis nature incipit feliciter. Amicum induit qui iustis amicorum petitionibus condescendit »..... (de lapide). F. 241 v, « Incipit magisterium Fratris HELYE Ordinis Minorum de elixiris ad album et rubeum. Cum de infrascriptis aquis, distillationibus, et dissolutionibus cum igne et sine igne » .....

The Clocks of Jacopo and Giovanni de' Dondi.

For a long time there was confusion, uncertainty and controversy as to the respective shares of JACOPO DE' DONDI, author of the *Liber aggregationis sive Aggregator Paduanus de medicinis simplicibus* (1), and his son, GIOVANNI DE' DONDI, in the construction of what appeared to be one of the first considerable mechanical clocks. Back in the last quarter of the eighteenth century, TIRABOSCHI (2), in his justly celebrated *Storia della Letteratura Italiana*, touched on the question and made use of facts concerning the career of JACOPO which had been communicated to him by a descendant, FRANCESCO SCIPIONE DONDI dall'Orologio. Thereafter the tendency was to hold that credit for the clock should go exclusively to GIOVANNI, that JACOPO was merely a medical man. But in 1896 ANDREA GLORIA, the learned historian and editor of the sources for the University of Padua during the thirteenth and fourteenth centuries (3), published an article on « The Two Marvelous Clocks Invented by JAMES and JOHN OF DONDI (4), » which seemed to give solid ground to stand upon. He showed that JACOPO was born at Padua before 1293, instead of in 1298, as had previously been stated, a date which made his election as municipal physician of Chioggia in 1313 come at the early age of fifteen or sixteen! GLORIA showed that he was recalled to Padua as professor in 1342, that he made a clock which was placed in

(1) This is the form of title used in an incunabulum edition in the British Museum numbered IC.666. The work was also called *Promptuarium medicinae*.

(2) Modena, 1772-1795 ; and in subsequent editions. For JACOPO DE' DONDI I have used the edition of 1823, vol. V, pp. 340-41.

(3) ANDREA GLORIA, *Monumenti della Università di Padova* (1222-1318), 1884, in *Memorie del Reale Istituto Veneto di Scienze, Lettere, ed Arti*, vol. 22. *Monumenti della Università di Padova* (1318-1405), 1888, in *Univ. Studi.*, vols: I-II.

(4) « I due orologi meravigliosi inventati da JACOPO e GIOVANNI DONDI, » in *Atti del Reale Istituto Veneto di Scienze, Lettere, ed Arti*, Serie 7. Tom. 7, Disp. 7, 1896.

the Carrara tower there in 1344, that he was called « dall'Orologio » in the documents — evidently because of his clock — and that this epithet, which became part of the family name thereafter, was not first applied to GIOVANNI. JACOPO's clock is not extant, but one in the Piazza del Signori, finished in 1434, is probably a copy of it and not of GIOVANNI's clock, on which he worked from 1348 to 1364, which had more elaborate astronomical details, which GIAN GALEAZZO placed in the castle at Pavia, and which also is no longer in existence. GLORIA regarded as contemporary and reliable the epitaph on the baptistery which alludes to JACOPO's clock and to his astronomical as well as medical knowledge (5). He further noted that PROSDOCIMO DE' BELDOMANDI, a mathematician and astrologer of the opening years of the fifteenth century, ascribed to JACOPO tables of the movements of the planets.

It has seemed advisable to rehearse here these main conclusions of GLORIA because they are totally ignored and the old assertions revamped in a recent paper by Dr. GIOVANNI ASTEGIANO entitled, « La Cittadinanza Veneta a Jacopo de' Dondi. (6). » So far as I can see, this paper brings to light no new facts to affect our estimate of JACOPO. It makes use of VINCENZO BELLEMO's *Jacopo e Giovanni de' Dondi dall' Orologio*, Chioggia, 1894, but makes no mention of GLORIA's article of 1896 and the data adduced by him. It repeatedly cites as authoritative the antiquated *Memoria* of FRANCESCO SCIPIONE MARCHESE DE' DONDI dall' Orologio, but does not mention TIRABOSCHI to whom it was communicated in the first instance. Indeed, ASTEGIANO states that JACOPO's work on tides remained unpublished until PAOLO RAVELLI reproduced it in the *Rivista Geografica*, vol. 19 (1912), from a manuscript of the early sixteenth century in the Ambrosian library at Milan, whereas both FABRICIUS and TIRABOSCHI list a sixteenth century edition of the work. (7). It is the more strange that ASTEGIANO

(5) « Ars medicina mihi coelumque et sidera nosse
Utraque nempe meis manent ars ornata libellis. »
Indeed, it would have been unusual for a medical professor of that period to have had no concern with astronomy.

(6) *Rivista di Storia delle Scienze Mediche e Naturali*, 16, 317-26, 1925.

(7) JACOBUS DE DONDIS, *De modo conficiendi salis ex aquis calidis Aponensibus et de fluxu et refluxu maris*, Venetiis, 1571. However, this edition is not given in the printed catalogues of the British Museum and the Bibiothèque Nationale, Paris.

did not know of GLORIA's article, since he cites a paper by VITTORINO LAZZARINI which appeared in 1916 in the same serial publication, *Atti del Reale Istituto Veneto di Scienze, Lettere, ed Arti.*

ASTEGIANO again asserts that JACOPO was only sixteen in 1314 ; he repeats the eighteenth century statement of FRANCESCO SCIPIONE MARCHESE DE' DONDI that JACOPO did not construct a clock and was unacquainted with mathematics and astronomy (although he wrote concerning the influence of the sun and moon on tides). Moreover, these discredited, or at least extremely questionable, statements are repeated, as if they were both assured facts and the outcome of new research into the subject, in the « Nineteenth Critical Bibliography » in *Isis*, 8, 744, 1926. This descriptive note also, probably by a misprint, since it gives the date 1364 for GIOVANNI's clock, gives the year of GIOVANNI DE' DONDI's death as the same as that of his father, JACOPO, in 1359, whereas he did not die until 1389. It has therefore seemed desirable to rectify the situation by calling the attention of the readers of *Isis* to GLORIA's conclusions.

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Ciruelo on the names "Arithmetical" and "Geometrical" Proportions and Progressions.

No history of mathematics offers explicit information on the question, why the adjectives « arithmetical » and « geometrical » came to be used in naming certain progressions and proportions. As far as the present writer has noticed, mathematical authors, both ancient and modern, are silent on this subject, except a noted Spaniard, PEDRO SÁNCHEZ CIRUELO, of the close of the fifteenth and the beginning of the sixteenth centuries. In his mathematical book, the *Cursus* (1), he explicitly asks the question, on the margin of one of his unnumbered pages, *Quare dicitur arithmetica*, and on the margin of the next page, *Quare dicitur geometrica*. In the text he answers the first query as follows : (2) « ... 2. 6 . 10 . 14 . 18. &c. This series of numbers they name in practical arithmetic... a progression of numbers. Such an aggregation of numbers is also called an arithmetical proportionality ; not that it alone holds in numbers and all others (3) in magnitudes. For indeed all species of proportionalities are found in numbers

(1) *Cursus quatuor Mathematicarũ Artiũ Liberaliũ : quas recollegit atqz correxit Magister* PETR' CIRUELUS *Darocisis Theologus simul et philosophus*. 1526. Tractatus secũdus secundi libri de pportionalitatibus, caput primum, d II. First edition Alcalá, 1516, also Paris 1516.

(2) ...2. 6. 10. 14. 18. &c. Hãc etiã numerorum seriẽ in arithmetica practica q̃ dr̃ algorismus vel abacus pgressionẽ numeror/ vocant. Dicĩ antẽ talis numeror/ aggregatio pportionalitas arithmetica : nõ q ipsa sola ĩ numeris cõsistat & aliae oẽs in magnitudinibus : qñquidẽ oẽs pportionalitatũ spẽs ĩ numeris oẽsqz ĩ magnitudinib' ĩueniant : sed dr̃ arithmetica quia solũ attẽdit numer/ vnitatũ excessus ĩter prĩcipales numeros q cõparanr̃. In geometrica vero portionalitate seq̃nte nõ curãtes de numero illarũ vitatum p quas prĩcipales numeri sese excedunt atqz excedunt' : principal 'r attẽdimus quota pars sit ille excessus respectu nũeri excedẽtis ve excessi : & hoc siue excessus ille sit vnitas siue binarı/ siue ternari' &c. vñ nõ immerito haec pportionalitas arithmetuica. i. numeratiua excessus dr̃ : sequẽs vero geometrica. i. mensuratiua eiusdẽ excessus. »

(3) Late Greek arithmeticians considered some ten different proportions

and all in magnitudes. But it is called arithmetical, because it deals merely with the number of units of excess in the principal numbers which are being compared. In geometrical proportionalities which follow later, not being concerned with the number of units by which the principal numbers exceed each other or are exceeded, we inquire principally *what part that excess* is of the number exceeding or exceeded. But here, whether this excess (4) be unity, or twice that, or thrice that, etc., this proportionality alone can properly be called arithmetical or numerative in its kind of excess ; similarly indeed, the one following is called geometrical in its kind of excess. »

On the next page Ciruelo repeats, that « geometrical or metrical » proportion, «like all others, is applied in arithmetical science, as well as in geometry, hence the adjectives arithmetical and geometrical in proportionalities are not named after the sciences in which they are used, but are due to the attitude towards the excess of one number over the other, an excess which we regard differently in one proportionality than we do in the other. » His explanation implies that the notion of difference of two numbers is more primarily arithmetical than is the notion of the ratio of two numbers ; the former certainly is more primitive, if we take counting genetically to preceed measuring.

Ciruelo's explanation is different from that which would naturally suggest itself to many of us, namely, that the terms « arithmetical » and « geometrical » were introduced, because they indicated the field (arithmetic or geometry) in which each of the proportions in question found its most specific application. It is precisely this explanation which Ciruelo rejects.

Looking into history, one finds that the Greek equivalents of the adjectives « arithmetical » and « geometrical » were not applied to proportions by Euclid, Apollonius of Perga, and Archimedes. These men used the term « proportion » (*ἀναλογία*) without any qualifying adjective, and they always meant by it our geometrical proportion. The phrasing used by Archimedes in the case of magnitudes in arithmetical progression is that the magnitudes « exceed each other by an equal (amount) », or they

(4) If a, b, c, d are in geometrical proportion, then a — b · b = c — d : d, or a —b : a = c — d : c.

are « magnitudes exceeding by the equal (difference). » (5) But there is evidence to show that the adjectives « geometrical », « arithmetical » and « harmonical » were used occasionally in connection with « means » and « proportions » before the time of EUCLID. NICOMACHUS who was an arithmetician of a late period in Greece states that the Pythagoreans, also PLATO and ARISTOTLE, dealt with arithmetical, geometrical and harmonical proportions (6). JAMBLICHUS (7), a still later writer, states that the Pythagoreans used three means, « the arithmetical, the geometrical, and a third which was known by the name of « subcontrary », but which ARCHYTAS and HIPPASUS designated as harmonical, since it appeared to include the ratios concerning harmony and melody. » Thus the name « harmonical » is pre-Euclidean. As to the names « arithmetical » and « geometrical », it might be claimed that they were not pre-Euclidean, and that NICOMACHUS and JAMBLICHUS used a terminology originating after the beginning of the Christian era to describe mathematical notions held by the Pythagoreans before the time of EUCLID. That such a contention is invalid follows from quotations from ARISTOTLE who uses the names « geometrical proportion » and « arithmetical proportion » in his *Nicomachean Ethics*, where he applies mathematics to the study of ethics (8). The distribution of justice implies four proportional terms. He defines a proportion A : B = C : D as an « equality of ratio », and says : « Mathematicians call this kind of proportion geometrical, for in geometrical proportion it comes to pass that the whole has the same ratio to the whole which each of the parts has to the other », i.e., that A + B : C + D = A : C. Further on ARISTOTLE explains that in corrective justice arithmetical proportion is observed. « But the just which exists in transactions is something equal, and the unjust something unequal, but not according to geometrical but arithmetical proportion... ; the law looks to the difference of the hurt alone. (9) »

(5) T. L. HEATH, *The Works of Archimedes*, 1897, p. CLXXXVI.

(6) NICOMACHUS OF GERASA, *Introduction to Arithmetic*, translated... by M.L D'OOGE, F. E. ROBBINS and L. C .KARPINSKI, New York, 1926, Chap. XXII, p. 265. See *Isis*, IX, 120-3).

(7) JAMBLICHUS, *In* NICOMACHI *arithmeticam introductionem liber*. Ed. H. PISTELLI 1894, p. 100.

(8) The *Nicomachean Ethics of* ARISTOTLE, translated by R. W. BROWNE, London, 1880, Bk. V, Chap. III, p. 125.

(9) *Loc. Cit.*, Bk. V, Chap. IV p. 126.

Here all doubt about the early use of the names « arithmetical » and « geometrical » proportion is removed, since these terms occur in a work doubtless written before EUCLID, certainly not after EUCLID. It is noteworthy that the statements in ARISTOTLE are in harmony with CIRUELO's contentions ; the name « geometrical proportion » was used because it referred to the *ratio of a part to the whole*, thus primary consideration being given to a metrical relation which occurs in arithmetic as well as in geometry. In Pythagorean time and, more generally, in pre-Euclidean time in Greece, arithmetic and geometry were not segregated ; they constituted one science. The word « geometrical » had reference, not to a branch of mathematics, but to the process of measurement.

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The Distinctive Features of Seventeenth Century Geometry.

Introductory Observations.

The modern period in education may be regarded as having its beginning (1) about 1600 A.D. To be sure, there is never an absolutely clear-cut beginning in such arbitrary divisions as this, but some well-defined characteristics may be noted which distinguish this period from any previous one. The general revival of learning had by this time come into its full power of operation. The study of the Humanities was rapidly being crowded out of first place by the pursuit of the physical sciences, and as the latter came into more and more prominence, not only were new directions of study marked out, but a general re-evaluation of the old studies took place. The putting of the methods of education on a sound scientific foundation was begun in the early seventeenth century.

Although modern psychology and pedagogy were then unknown, it is found that a study of the contents of the texts of that period reveals many suggestions of, and valuable similarities to, modern instruction, both as to method and curriculum. Despite the pre valent belief that the lecture method was the only one of any consequence then in use, a careful research into the matter shows that the seventeenth century shares greatly in the formulation of many fundamental ideas in modern class-room pedagogy. There is deep significance in the words of ADAMSON (2) : « Not a few of the conceptions, small as well as great, which we are apt to consider characteristic of our own, or of the generation or two immediately senior to us, are but re-statements of principles and

(1) S. S. LAURIE : « Studies in the History of Educational Opinion from the Renaissance », Cambridge, 1905, p. 119. (Hereafter referred to as LAURIE)

(2) JOHN WILLIAM ADAMSON : « Pioneers of Modern Education », Cambridge 1905, p. V.)

devices which took their earliest modern shape in that same seventeenth century ».

Revival of Interest in Geometry.

Along with this general revival of learning came a new impetus to the interest shown in geometry. (3) First among the causes of this is to be mentioned, of course, the general awakening already referred to, along with which all things were more or less carried. But in an indirect or subsidiary way other factors entered very effectively. (1) The possibilities that lay in the new process of printing had by this time become widely comprehended and already millions (4) of volumes had been printed. Thus within a century and a half after the invention of the press every important city of all the leading countries had at least one expert printer. (2) At the beginning of the sixteenth century Latin was still the leading language of the learned. But during that century the vernacular became increasingly more popular, and by the time of its close much of the work was printed in the language of the land. (3) A great change was taking place in the universities (5). From their beginning as groups of organized scholars they had been befriended by the Church — for learning had, up to that time, been almost entirely of the priesthood — and early came under the Church's control. Throughout the Middle Ages, therefore, they were ever at the service of the Church, offering only those methods sanctioned by it. The doctrinal struggles of the Church were the outstanding problems of the universities, and sometimes came near disrupting them. Questions such as that of Nominalism and Realism were often carried to the point of unprofitable controversy. Even throughout the greater part of the seventeenth century the University of Paris was heavily burdened by these inner struggles. Consequently reforms in education did not come from within the universities. They were slow to cast off their medievalism. But one by one this did come to pass. A great expansion took place ; the number of students increased

(3) In this work the term geometry will be used for elementary geometry.

(4) ROBINSON and BREASTED : « History of Europe Ancient and Medieval », GINN & Co., 1920, p. 484.

(5) See RASHDALL : « The Universities of Europe in the Middle Ages », Oxford, 1895, 2 vol. ; also LAURIE, chap. IV.)

markedly ; a wider range of courses was offered ; new departments were organized ; and a greater independence was shown in the instruction. Furthermore, many new universities were founded, as, for examples, the universities of Franeker in 1585, of Harderwijk in 1600, of Groningen in 1614, of Utrecht in 1634, of Budapest in 1635, of Lund in 1666, and many others.

A fourth factor in bringing about this revival of interest in geometry is the practical need of a knowledge of the subject for the study of warfare, surveying, navigation, astronomy and other interests then widely pursued. Experimental science received in the seventeenth century its first marked attention. (6) The inductive method of investigation was coming into its own. Chemistry took the place of alchemy ; medical science supplanted magic and mysticism ; and theories were tested, if not actually formulated, by numerous experiments. The first scientific instruments made their appearance. The telescope (1609), thermometer (1597), barometer (1643), microscope (1650) and many others were invented. Furthermore, scientific societies and academies, such as « Academia Secretorum Naturae » (1560) at Naples, « Accademia dei Lincei » (1603) at Rome, the Royal Society of London (1662) and the French Academy (1666), were organized, with the business of discoursing and considering « of philosophical enquiries, and such as related thereunto : — as Physick, Anatomy, Geometry, Astronomy, Navigation, Staticks, Magneticks, Chymicks, Mechanicks, and Natural Experiments ; with the state of these studies and their cultivation at home and abroad » (7). Obviously the most of the above interests called for a knowledge of at least the practical side of geometry.

From the causes thus far named definite results were certain to follow. First to be noted is the great productive activity. Aside from the widening out of the subject into less elementary geometry through the works of men like DESARGUES (8), DESCARTES (9), and PASCAL (10), numerous important translations and editions (11)

(6) SEDGWICK and TYLER : «A Short History of Science», New York, 1917, p. 259

(7) IBID., p. 269

(8) GÉRARD DESARGUES : « Traité de la perspective », 1636.

(9) RENÉ DESCARTES : « La Géométrie », 1637.

(10) BLAISE PASCAL : « Essai pour les coniques », 1640.

(11) T. L. HEATH : « The Thirteen Books of EUCLID 's Elements », Cambridge, 1908, 3 vol. vol. 1, p. 104 ff.

of EUCLID were printed. The authors of some of these added explanatory remarks (12) and observations which make their work of great importance in a consideration of the teaching of the subject. In addition, moreover, to the many translations of EUCLID, many other texts on geometry appeared which differ somewhat in material presented, and vastly more in the symbolism used and in the manner of presentation of the subject-matter. These points will be considered more fully later. Besides translations of EUCLID's Elements, nearly all of the other important texts appeared in several languages, thus making the study for the first time easy of access. Thus the works of the Jesuit PARDIES (13) appeared first in French, was translated into Latin and English, and went through many editions in each language.

This readier availability of texts, made possible by the cheapening of the art of printing, had two notable effects. First, it made the study of mathematics wide-spread and popular. During the seventeenth century geometry was taken up in most of the universities not already offering it, and in some of the gymnasia of Germany, and the number of secondary schools giving it consideration was increasing (14). Chairs of mathematics in the universities were established in greater numbers and a knowledge of EUCLID was required for the degree in many. Numerous private teachers offered their services to those desiring training in mathematics, using texts which, by their attractive titles or otherwise, advertised the subject. Thus, among the earliest of these may be noted ROBERT RECORDE's (15) « The pathwaie to knowledge, containing the first principles of Geometrie, as thei maie moste aptly bee applied unto practise, both for use of instrumentes Geometricall, and Astronomicall : and also for protection of plattes in every kinde, and therefore much necessarie for all sortes of menne. » Picturing the subject as a « gloriously beautiful geometrical

(12) D. E. SMITH: « History of Mathematics », GINN and Co., 1923-25, 2 vol., vol. 1, p. 335, also M. CANTOR : « Vorlesungen über Geschichte der Mathematik », 3 vol., vol. 2, p. 556.

(13) IGNACE GASTON PARDIES : « Éléments de Géométrie », Paris, first edition 1671, 12 mo.

(14) ALVA WALKER STAMPER: «A History of the Teaching of Elementary Geometry » New-York, 1909, p. 65. Hereafter referred to as STAMPER.

(15) This is the title of the 1574 edition. The first edition was printed in London in 1551.

gallery », BEUTEL (16) makes his appeal. Others emphasize the ease with which the subject can be mastered from their texts.

The greater supply of texts also opened the way for the discovery and development of new methods of teaching. For it is clear that a lack of books would have a limiting effect upon the manner of instruction. The lecture method had fitted into this handicap most satisfactorily and hence had been commonly used. While lecturing continued to predominate throughout the seventeenth century, the contents of the geometries reveal other methods coming into use. The theories of pioneers in the field of education were given serious consideration. In Germany WOLFGANG RATKE (1571-1635) emphasized, in 1613, the use of the vernacular, the presentation of material in such manner that it would not be beyond the student, and advocated other changes in teaching. Although in advance of his time he was taken seriously enough to be given the opportunity of setting in order the administration of the schools in several German cities, in which, however, he failed, more, perhaps, because of his weakness in the execution of his plans than because of the lack of soundness of his principles. Later we find the educationist COMENIUS setting « forth a scheme for readily and soundly teaching everybody everything » (17) advocating the teaching of science, along with many other subjects in the schools. He was among the first to stress the psychological above the logical in instruction. In England, FRANCIS BACON (1561-1626) was propounding his ideas which suggest much of what we have since found good. In « De Augmentis » he discussed the mode of scientific inquiry in such a way as to make it quite clear that he saw the distinction between what we now term the logical and the psychological methods of attack. He furthermore states that the teacher who teaches his pupil after the same fashion in which that knowledge grew in his own mind is to be preferred.

These educational changes were applied to the teaching of geometry (18) as well as to all other subjects of the curriculum. But an investigation of the text-books used shows — as might be

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(16) « Eine herrliche schoene Geometrische Gallerie », etc. First edition, 1660. See the author's checklist in *Isis*, 10, 21-32, for full title page.

(17) J. A. COMENII Opera Didactica Omnia, Amsterdam, 1657, part I, col. 442.

(18) W. H. PAYNE's tr. of COMPAYRÉ : « The History of Pedagogy », Boston, 1899 p. 126 ; also, however, STAMPER, p. 63.

expected — that these pioneer ideas found their way into actual teaching only very slowly.

As a final result to be mentioned, attention must be called to the extremely practical nature of the geometries produced during this period. Whereas EUCLID was painstaking in his efforts to avoid anything practical, even any trace of mensuration, we find the writers of the seventeenth century stressing the utility side in the prefaces of their books, writing it in the titles, illustrating it in the drawings of their texts, describing the use of instruments other than the ruler and compass, and binding their works on geometry up with treatises on cosmography, fortifications, navigation and the like. This was sometimes carried on even to the extent of leaving out the rigorous demonstrations of theorems entirely, with no remarks whatever. More often, however, reference was made in each case to the proposition and book of EUCLID where the proof was to be found. But numerous other geometries were meant to be complete in themselves. This tendency toward the practical and away from EUCLID is the result of the fourth factor mentioned above as a cause of the revival of interest in geometry. To some extent, at least, it must have been due to the pressure from the universities, then being felt in the secondary schools, making for the shifting of geometry from the former to the latter. EUCLID was too difficult, and consequently other texts were written, (19) containing less of rigor and more of the practical. Not much, however, can be said with authority as to the weight of this last point.

Necessary Precautions.

At the outset of an investigation of the teaching of geometry of the past it must be born in mind that we are always viewing things in the light of the present with our modern points of view. Care must be taken not to read into the past that which is not there, and, likewise, not to fail to consider the circumstances then involved.

With these things in mind, let us remind ourselves of what is perhaps well known, that seventeenth century geometry was meant for, and used by, adults. While the shift of the subject

(19) STAMPER, p. 64.

from university to secondary school began early in this period in some places, it was not until the close of the eighteenth century that geometry was generally taught in the German, French and Swiss secondary schools, and not until well into the nineteenth century that the teaching of geometry gained a foothold in the secondary schools of England, Russia and Holland (20). In fact, even in Cambridge university, there were no courses offered in mathematics at all as late as 1632 (21). And in the United States geometry was not made a college entrance requirement until 1843 or 1844 (22), and then but little knowledge of the subject was needed. This means that whatever secondary schools there were had been teaching but very little geometry. Some of the academies taught the subject before this time, but they were then above preparatory school rank in the courses which they offered.

Hand in hand with the shift of geometry as a subject of study from the older to the younger student were developed the new ideas of choosing the material and arranging its sequence in such order that the pupil might proceed from the less difficult to the more difficult, at the expense, if necessary, of rigorous logic. All other things considered as equal, then, the teaching in the seventeenth and present centuries should still be different, both as to material used and as to method of presentation.

It is also to be observed that almost the sole source of material available for an investigation of this character is found in the text-books themselves. Very little else has been left us. These original sources as given in the checklist previously published by the author in *Isis* must be carefully examined if a true conception is to be obtained. Such an investigation reveals many valuable contrasts and similarities to which the attention of the reader will now be called.

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The similarity of the geometries of the present day to the Elements of EUCLID is sufficiently marked to warrant the contention that the work of EUCLID has always been to a more or less degree the point of departure in the writing of geometry texts. Upon

(20) STAMPER, pp. 70, 73, 96, 90, 92, 94.

(21) FLORIAN CAJORI : « The teaching and History of Mathematics in the United States », Washington, 1890, p. 20.

(22) STAMPER, p. 97.

examination we find that the great majority of seventeenth century geometries support this position. Aside from the many purely practical works, both the body and the order of material follow generally EUCLID's work. But the departure is in many cases quite wide. Definitions are changed ; different proofs appear ; new material is introduced ; and there is a clearly discerned shift in the chief value of the subject.

Definitions.

The word «geometry » itself is usually discussed in the prefaces, which are as a rule elaborately drawn out. « The art or science of measuring » is often offered as a definition, but authors usually deplore the fact — fully recognized by the Greeks — that the name falls so far short of expressing the full significance of the subject. Thus in the preface of JOHN DEE (23) we read : « This Science of Magnitude, his properties, conditions, and appurtenances : commonly, now is, and from the beginning, hath of all Philosophers, been called Geometry. But verily, with a name too base and scant, for a Science of such dignity and amplenesse. »

An increasing number of works give the definitions of « point », « line », « surface » and « body » in the reverse order from that given by EUCLID. This, originally done by GERBERT (24) is now generally recognized as the more pedagogical presentation. Examples of this are found in the anonymous work, « The Elements or Principles of Geometrie (25) at the beginning of Chapter I : « Geometrie is the Art of Measuring Bodies : in order to which end, it considers three principal that belong to a Bodie, namely, Superficie or Surface, Lines, and Points : Which in their proper notion, do signifie, the Out-side, the Edg, and the Corners of a Bodie ; and so, are real parts of it : But in the sense that Geometrie uses them, they do not properly belong to a Bodie, only are applied to it by our imagination. For if a Bodie be dipt in water, though the water touches it everywhere, yet we may fancy a division between it and the water, which imaginary division

(23) Found for example, in BILLINGSLEY's. « The elements of Geometrie--- », 1570. For a fuller description of the works referred to hereafter, see the author's checklist in *Isis*, 10, 21-32.

(24) JOHANNES TROPFKE : « Geschichte der Elementar-Mathematik », v. IV, p. 29. Hereafter referred to as TROPFKE : « Geschichte ».

(25) Two editions appeared in 1684.

is called a Surface. And thus, every imaginary division of a Bodie, begets a Surface ; (between the parts so divided :) of a Surface, begets a Line ; of a Line begets a Point. And since whatsoever divides a thing terminates or bounds the parts so divided ; therefore, 1. A Point is said to be bound of a Line. 2. A Line the bound of a Surface. And, 3. A Surface the bound of a Bodie. The first, having neither length nor breadth ; The second, length without breadth ; and the third, length and breadth without depth. »

It is interesting to note here that the beginning of an anonymous set of « Geometry notes of 1726/7 (January) », found in the library of the Historical Society of Pennsylvania, Philadelphia, has exactly the above words, with the exception of the change of the spelling of «Geometrie » to « Geometry ». This suggests that the anonymous text above referred to was one of the earliest printed works on geometry used in the United States (26).

The second edition of DE GRAAF's « Inleyding tot de Wiskunst » begins also with the body, « Lichaam », and thus leads to the definition of point, « Stip » (27). He then adds that this is the only magnitude found in nature, the others being obtained by considering divisions of the first. The Euclidean definition of a surface, « Vlak (Superficies) », is first given, and after it he states, « or is the boundary of a solid » (28). Likewise the line is the boundary of a surface, and the point of a line.

The order of definitions given by LAMY (29) is : solid, point, line and surface.

On the other hand, quite a few of the works give the same definitions as are found in EUCLID's Elements, but add the ideas of others and of their own to give clearness. An example of this is found in HOFMANN's work (30), page 3 : « A point is that which is indivisible. — Although the point has, according to the above definition, no size, it nevertheless is a beginning or origin of the same. For through the movement of the point in the length a line is formed, and through the movement of a line

(26) Compare STAMPER, p. 97.

(27) 2nd. ed., 1706, p. 4, « Lichaam (Corpus) is een Grootheit van drie afmetingen, hebbende Lengte Breete en Diepte. »

(28) *Ib.* p. 4, « of is het buytenste van een Lichaam ».

(29) « Les Éléments de Géométrie » 1685, p. 1.

(30) Teutscher Euclides » 1653.

in the breadth a surface is formed, and through the movement of a surface in the height a body is formed, and thus from nothing comes something. » (31). We see in this passage the idea of generation, not mentioned in EUCLID's Elements, but first appearing in the works of HERON OF ALEXANDRIA.

Similarly, while there is a predominance of the way of defining the straight line as given by EUCLID (32), there is often found either a change of definition or some addition to clarify the statement. Thus LAMY (33) gives the one statement, « The line or length which is the shortest between two points, is a straight line. » (La ligne ou la longueur qui est la plus courte entre deux points, s'appelle ligne droite.) HOFMANN (34) mentions several ways of looking at it : « Eine gerade Lini/ linea recta, ist/ so zwischen den Endpuncten allenthalben gleich in liget. Denn solche wird verstanden/ wenn das fliessende Punct von einem Ort zum andern schnur gleich zu/ den kürtzesten Weg geführet wird. Dem Platoni ist die gerade Lini/ dessen innere oder Mittelpuncten die eussern oder Endpuncten bedecken. Andern aber/ dessen Mittelpuncten den eussern oder Endpuncten widerstehen ; oder dessen Mittelpuncten von den eussern nicht abwerts treten/ noch die eussern von den innern/ oder den Mittelpuncten. »

Various ways of expressing the idea of angle are used. Besides the definition of the angle as the inclination between two lines we find RECORDE (35) defining it as follows : « Therefore, when so ever you doe see any formes of lines, to touche at one notable pricke — then shall you not call it one crooked line, but rather twoo lines ; — when so ever any suche meetyng of lines doeth happe, the place of their metyng is called an angle or corner. » The anonymous « Elements or Principles of Geometrie », of 1684, also defines the angle as corner. METIUS (36) arrives at his definition only after he has first discussed the perpendicular — rather a

(31) « Ein Punct ist/ so da unzertheilig ist. - - - Obwol das Punct vor sich nach dem vorgehenden Elem. keine Groesse hat/ ist es doch ein Anfang oder Ursprung derselben. Denn durch Fortführen des Puncten in die Lenge/ wird eine Lini/ und durch Bewegung der Lini in die Breite/ wird eine Fleche und durch Bewegung der Fleche in die Höhe/ wird ein Corpus/ und also gleichsam aus Nichts etwas. »

(32) « A right line is that which lies evenly between its extremities. »

(33) « Eléments de Géométrie », 1685, p. 3.

(34) « Teutscher Euclides, » 1653, p. 4.

(35) « The pathwaie to knowledge », 1574, p. 2 unnumb.

(36) « Manuale-Geometriæ Practicæ », 1646, p. 85.

round-about way — thus defining right angle, and, from that, getting at the general definition.

A third group of writers, going back to APOLLONIUS (37), bring out the surface idea in their consideration of the plane angle as that portion of the plane comprised between two given intersecting lines. Thus, as an example, LAMY (38) says: « Une surface plane comprise entre deux lignes droites qui se joignent en un point, & qui se coupent étans prolongées, se nomme Angle plan. » The same idea is brought out in the works of HOFMANN (39) and ARNAULD (40).

Finally, the notion of the turning of the sides about the vertex of the angle is brought out. The conception goes back to SCHMID's work of 1539 (41) which was the first to make use of a circle with center at the vertex of the angle. The size of the angle was determined by the length of the arc between its sides. This same thought appears in the work of PARDIES (42) : « Prenez deux lignes ab & ac, qui se touchent en a : si vous imaginez que ses deux lignes s'ouvrent comme un compas, en sorte qu'elles demeurent toujours attachées en a comme par le clou du compas, tandis que l'extrêmité s'écarte de l'extrêmité b ; alors vous concevrez que plus ces extrêmités s'éloigneront mutuellement, plus aussi se fera grand l'angle qui est entre deux ; & au contraire, si vous approchez davantage ces extrêmités, vous ferez que les lignes seront plus inclinées, ou plus panchées l'une vers l'autre et l'angle sera plus petit. »

Symbols.

A very remarkable change is to be noted in the use of symbols. While geometry does not as readily as algebra lend itself to the use of symbols, there are certain words which, because they recur so often, can expediently be represented symbolically. This fact was recognized very early in the history of the subject. But each writer devised his own characters. No approach to the standardizing of characters was gained until the seventeenth century. Then many of the symbols of algebra were carried over into works on geome-

(37) TROPFKE : « Geschichte », v. IV, p. 46.
(38) « Eléments de Géométrie », 1685, p. 41.
(39) « Teutscher Euclides », 1653
(40) « Nouveaux Eléments de Géométrie », 1683.
(41) TROPFKE : « Geschichte », v. IV, p. 47.
(42) « Elementa Geometriæ », 1671. This quotation is from the 1725 edition, p. 4.

try and others came into common use directly from these geometry texts. One of the richest works from the point of view of the number used is the book of HERIGONE (43) at the beginning of which he uses twelve pages to explain his system of notation. Besides numerous abbreviations, he gives a considerably longer list of symbols than is commonly recommended today (44). He seems to have taken pleasure in devising as many as possible, even though, in some cases, no advantage was gained by their introduction, To illustrate, he uses U for « vel », « ou », « or » ; ꝓ for « ad » « à », « to » ; and ꝯ for « in », « en ». Some of the few symbols which are used in the same sense as we now use them are + for « plus » ; ⊥ for « perpendicular » ; ∠ for « angle » (but he also gives < for « angle ») ; ⊙ for « circle » ; △ for « triangle » ; □ for « square » ; and ▭ for « rectangle ». On the other hand, for « minus he gives ~ ; for « straight line », — ; for « right angle » ⌟ ; for « parallel » = ; for « equals » 2/2 ; for « is greater than » 3/2 ; for is less than» 2/3 ; for « pentagon », « hexagon », etc., 5 <, 6 <, etc. ; and « a segment of a circle » is represented by a D tipped over, either to the right or left.

Propositions, definitions, etc. are stated in both Latin and French. The following proposition from Book I, page 16, will serve to illustrate his method of presentation :

Si duae rectae lineae se mutuo secuerint, angulos ad verticem aequales inter se efficient.

	∠ aed 2/2 ∠ ceb.
	Demonstr.
13. 1.	∠ aec × ceb 2/2 2
13. 1.	∠ deb × ceb 2/2 2
	ceb commun. substr.
3. a,1 d.	∠ aec 2/2 ∠ deb, ∝ ∠ aed 2/2 ∠ ceb.

Hypoth.
ab & cd snt ———.
Req. ▯ demonstr.
∠ aec 2/2 ∠ deb,

(43) « Cursus Mathematicus », 1634.

(44) See « The Reorganization of Mathematics in Secondary Education », 1923, p. 77. Hereafter referred to as « Reorganization of Mathematics ».

Si deux lignes droictes se couppent l'une l'autre, elles feront les angles au sommet éqaux entr'eux.

Notice in the above proposition the similarity to our present divisions of the demonstration. In construction problems HERIGONE's work has « Hypoth. », « Req. fa. », « Constr. » and « Demonstr. » for our « Given », « Required », « Construction » and «Proof». This similarity is not characteristic of HERIGONE's work alone, however, for in SIMON STEVINS' work, « Les Œuvres Mathématiques » (45), we find equally clear divisions of the demonstration. In this work one finds the following arrangements : « Le donné », « Le requis », « Demonstration », and « Conclusion ». Or, in construction problems : « Le donné », « Le requis », « Construction », « Conséquence » (sometimes), and « Conclusion »,

But before HERIGONE's work appeared, the most of the commonest symbols now employed had already been used by someone. Our + and — signs first appeared in print in 1489, in a work by WIDMANN (46) ; and ROBERT RECORDE's « The Whetstone of Witte », 1557, contains the first modern equality sign, although as late as 1706 NAUDÉ (47) used· The now common symbol ∴ for « therefore » is first found in J.H. RAHN's « Teutsche Algebra », of 1659. (48) (The characters for « similar to » and for « congruent » were introduced by LEIBNIZ in 1679. (49). WILLIAM OUGHTRED's « Clavis Mathematicae » (50) contains the first use of the modern sign of multiplication. In the 1652 edition, page 11 we read : « Ut si latus AB + CD multiplicandum sit in se, producetur quadratum ABq + 2 AB Dd + Cdq. » (If the side AB + CD is multiplied into itself the reis produced the square $\overline{AB}^2 + 2$ AB.CD + $\overline{CD}^2$.) Instead of OUGHTRED's ABq for our $\overline{AB}^2$, HERIGONE wrote the more nearly modern form AB^2.

SIMON STEVIN (51) in his solution of the triangle when two sides and an angle adjacent to one of them are given, thus yielding

(45) « Œuvres», 1634.

(46) STAMPER, note, p. 58.

(47) « Grunde der Messkunst ».

(48) Pp. 52, 53. « Nach art der analogien ist das product von beyden aussern stellen gleich dem product beyder mittlern stellen : welches allhie mit den 3. punctlinen .˙. so da heissen ergo oder deszhalben ; bemerket wirt ».

(49) TROPFKE : « Geschichte », v. IV. p. 19.

(50) First ed., 1631

(51) « Œuvres », 1634, v. II. « La doctrine des Triangles, La Géographie, & L'astronomie », p. 15.

as a rule two correct solutions for the third side, uses the same letter for the two possible positions of the unknown vertex, with one dot over it in the one position and two dots over it in the other. This is one of the earliest indications of the use of indices, which appeared in their modern form in the works of LEIBNIZ. (52)

Although the symbolism of geometry had not even at the end of the seventeenth century become fixed, it becomes clear from the foregoing paragraphs that the commoner symbols had by its close met with at least general acceptance and a more or less systematic use.

Terminology.

But if this be true of the symbolism it is not so of the terminology, especially in the English works that make any attempt at a departure from EUCLID. ROBERT RECORDE, the earliest English writer on geometry, uses for « point » the alternate word « pricke », which occurs more often than does the usual « point ». For « angle » he also uses « corner », for a plane he gives the expression « plain plat », and defines it as « that whiche is made all equall in height, so that the middle partes, neither bulke up, neither shrinke doune more then the bothe endes ». (53) For a cube he uses the word « dye » or « ashler », and for a sphere, « globe ». Considerably more like our present terminology is the BILLINGSLEY edition of EUCLID's Elements (54), which contains many of our terms (55), and the anonymous work, already referred to, on « The Elements or Principles of Geometrie » of 1684. But in the latter also, we find numerous examples of unstable terminology. The following axioms numbers 10 and 14, page 17 (unnumb.) will serve to illustrate :

« 10. Those things that have the same Reason to a third thing, (or to things that are equal,) are themselves equal : That is, those things that are equally great in respect of another, are equal between themselves ; and convertedly.

14. A Whole, and all its parts together, have the same Reason to a third thing. »

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(52) TROPFKE : « Geschichte », v. IV, p. 15.

(53) « The pathwaie to knowledge », 1574, p. 4 unnumb. of defs,

(54) Dated 1570.

(55) KARPINSKI and FIEDLER : « The Terminology of Elementary Geometry in the Feb., 1924, issue of *School Science and Mathematics*.

Even in the early eighteenth century there is plentiful evidence of this unsettled terminology. Thus, in WELLS' « The Young Gentleman's Arithmetick, and Geometry » (56) is found the following :

« The mutual Inclination of two Lines meeting together, is call'd an Angle. And the Lines thus metting together, are call'd the Legs of the Angle. And the Point, wherein they meet, is call'd the Vertex or Head of the Angle, or the angular Point. » Again, on page 186, he has : « An Oblong is that, which has four right Angles, but only the two opposite sides equal. » In a footnote on the oblong, he says : « It is call'd by common Workmen, a Long-square ; and by Mathematicians frequently a Rectangle from its four right Angles. »

The English work of WILLIAM LEYBOURN (57) is very similar to the texts of the present time from the point of view of terminology. With the exception of quite a few terms now no longer in use, such as orthigonium (right triangle), ambligonium (obtuse triangle) and oxigonium (acute triangle), his work is quite similar in its use of terms to modern texts. These seventeenth century writers had much influence in bringing about the great predominance of Latin terms which finally came through to the present time. (58)

Attempts at a stabilized terminology met likewise with poor success in the works written in other languages. The eighteenth century was well in the past before any appreciable indication of relief appeared. Some writers sought to solve the difficulty by giving, in effect, a short mathematical dictionary at the beginning of their work.An example of this is found in J. VAERMAN's « Academia Mathematica of Oeffen School vande Wis-konst » (59). Works like RALPHSON's (60) and the « Lexicon Mathematicum » of HIERONYMO VITALI (61) with its more than a thousand pages give an idea of the magnitude of the task.

The word «radius» is an interesting one to follow from its first

(56) « The Young Gentleman's Aritmetick, and Geometry » 1723, edition, p. 182. First ed. 1713.

(57) « Cursus Mathematicus », 1690, defs.

(58) More than half of the technical terms of elementary geometry are from the Latin. See KARPINSKI and FIEDLER: « The Terminology of Elementary Geometry » in *School Science and Mathematics*, Feb. 1924.

(59) Edition of 1720, p. 2.

(60) « A Mathematical Dictionary », 1702.

(61) Second ed., 1690.

introduction to the time when it became a fixed term of geometry. It was introduced by PETER RAMUS in « Scholarum Mathematitarum » in 1569 (62), was used by some writers and ignored by others all through the seventeenth century. Thus MAROLOIS (63) refers to it as « Raid » ; METIUS (64) and PARDIES (65) use « Radius » ; while the word « rayon » is found in the works of LAMY (66), Le CLERC (67) and BLONDEL (68). On the other hand STEVIN (69), RUDD (70), ARDÜSER (71) and MÜLLER (72) prefer some other term in its stead. But by the end of the century the word was well on the way to universal acceptance.

Numerous other terms, such as « horizontal », introduced by ARDÜSER (73) in 1627, and « supplement », first defined in our modern sense by HERIGONE (74) in 1634, had their beginnings and gradually came into common use during the seventeenth century. This is a period of great importance in the study of the technical terms of geometry.

Figures.

The use of figures and the custom of designating their vertices by means of letters were established long before the period in which we are chiefly interested. But seventeenth century texts are far from uniform in the matter. The most of them have, according to our present standards, a moderate number of figures.

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(62) « TROPFKE : Geschichte », v. IV, p. 108. Also SMITH : « History of Mathematics », v. II, p. 279.

(63) « Opera Mathematica », 1628, p. 3,

(64) « Manuale Geometriæ Practicæ, 1646, p. 84.

(65) « Elementa Geometriæ », 1711, 1st. ed. 1671.

(66) « Les Eléments de Géométrie », 1685.

(67) « Traité de Géométrie », 1690.

(68) « Cours de Mathématique », 1699.

(69) « Œuvres 1634.

(70) « Practical Geometry », 1650.

(71) « Geometriæ, Theoriæ et Practicæ», 1627 TROPFKES «Geschichte», v. IV, p. 109. states that ARDUSER uses « radius » throughout. This is incorrect. On fol. 1 and 2, where terms pertaining to the circle are defined, no mention is made of either radius or semidiameter. On fol. 29 « semidiameter » is used, while « halben diameter » is found on fol. 135, 136, 137 and elsewhere.

(72) « Geometria Compendiaria Quadripartita » 1706.

(73) TROPFKE : « Geschichte », v. IV, p. 50.

(74) Ibid. p. 52.

The productions of METIUS (75), MAROLOIS (76) and MÜLLER (77) are examples of those in which figures are very plentiful. On the other hand the number is quite scarce in the writings of FINAEUS (78), RAMUS (79) and ALSTED (80).

In some of the productions, as in the « Scala Grimaldelli Libro di Aritmetica e Geometria » of FELICIANO (81) and the « Epitome » of BOUELLES (82), all the figures are placed in the margin. In others they are inserted on separate sheets throughout the book. The « Cursus Mathematicus » of Leybourn and the « Geometrie » of MAROLOIS (84) have this arrangement. A third group of works, among which are those of MÜLLER, (85) WOLFF (86) and TACQUET (87), have all sheets containing figures inserted in the back. Finally, many of the texts have their figures appearing along with the context, as is common in our modern works. As good examples of the last group the writings of VAN CEULEN (88), DE CHALES (89) and ARDÜSER (90) may be mentioned. The 1646 edition of ARDÜSER's work, which contains two books more than the earlier edition, has the figures for the first twelve books printed along with the context and those for the last two inserted at the back.

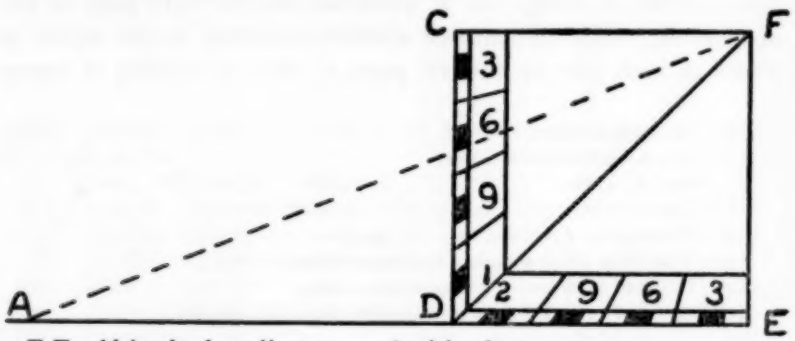
Great care is shown in the construction of figures in some of the books. Those in GRUBER's work (91) are small but very neat. The Latin edition of LE CLERC, « Nova Geometria », (92) has excellent drawings, all of which are on the right page of the open book, while the printed matter pertaining to the figure is found in each case on the left page, so that no turning of leaves

(75) « Manuale », 1646.
(76) « Opera Mathematica », 1627.
(77) First ed., 1706.
(78) « Opere », 1587.
(79) « Arithmeticæ Libri duo : - - - », 1599.
(80) « Methodus admirandorum mathematicorum », 1641.
(81) « Libro di Arithmetica e Geometria », 1669.
(82) Edition of 1514. Editions running into the 17 th. century.
(83) « Cursus Mathematicus », 1690.
(84) « Opera Mathematica », 1627.
(85) Op. cit., 1706.
(86) « Elementa Matheseos Universae », 1717.
(87) « Opera Mathematica », 1669.
(88) « De arithmetische en geometrische fondamenten », 1615.
(89) « Mundus Mathematicus », 1690.
(90) «Geometriæ, Theoricæ et Practicæ », 1627.
(91) Second ed., 1702.
(92) Amsterdam, 1692.

is necessary for the study of any proposition. The MAROLOIS-GIRARD work (93) also has unusually well-drawn figures, and the propositions are numbered according to the number of the figure and the plate on which it is found. Some of the more practically inclined writers make a special point of having drawings exceptionally elaborate and attractive. POMODORO produced a very interesting volume (94) on the use of mathematical instruments. With every page of printed matter there is a page of drawings illustrating the various constructions and calculations. An inspection of the « Mathesis Juvenilis » of STURM (95) shows at once that even the figures help to bring out the emphasis on the practical. Another remarkable little book in this respect is « De quadrante geometrico libellus, » by CORNELIUS DE JUDEIS (96). The author is nowhere satisfied with a plain geometric figure. On page 37 he shows how to determine, by means of the « geometric square », the width of a trench (fossae). He pictures the trench with water in it, geese swimming about, a man making use of the « square », and two men on the opposite bank fishing. In the figure, AE= width of trench and the eye of the observer is at F. Below the drawing is the « schema » :

C,B. punctum a regula tactum est, — 5 altitudo.

C,F. Tota scala est, — 12 distantia.



E,F. Altitudo baculi est, — 6 altitudo.

Iam ita colloca.

5—12—6. Facit 14 2/5 latitudo E,A. »

(93) « Opera Mathematica », 1627.
(94) « La geometria prattica », 1623.
(95) « Elementa geometriæ », 1699.
(96) « De quadrante geometrico libellus », 1594.

With some exceptions capital letters are used in connection with the figures. The 1599 edition of RAMUS (97) does not however, use capitals ; where letters are used, they are lower-case. In some of his figures, especially those in connection with problems of mensuration, numerals alone are found. FELICIANO (98) uses capitals together with numerals, or numerals alone. No system at all seems to have been used by METIUS (99). Both small and large letters are used to designate vertices, and sometimes some of each kind are found in the same figure (100). Usually, however, he holds either to the upper or lower case letters in any one figure (101). In some figures the letters do not stand upright when the book is held in reading position (102). Still another irregularity is seen in the fact that capital letters are of different sizes in the same figure. Thus, on page 113 of his work is found a figure with four points of intersection lettered with capitals, but the A is more than twice the height of either of the other three letters.

Occasional use is made of Greek letters (103). But in fully three-fourths of the works of this period capitals have the preference. There is as yet no indication of the use of the capitals for vertices, small letters for the opposite sides and Greek letters for the corresponding angles (104), as is now recommended (105).

Axioms and Postulates.

In general, seventeenth century geometries are rather careless about preserving the clear-cut distinction between definitions and postulates and between postulates and axioms. As a rule, only those books which adhere quite closely to EUCLID's text have anything at all to say about postulates. The majority of practical works either include both axioms and postulates in the definitions,

(97) « Arithmeticæ Libri duo ».

(98) « Libro di Arithmetica e Geometria », 1669.

(99) « Manuale », 1646.

100) *Ibid.*, p. 361.

(101) *Ibid.*, pp. 161, 133.

(102) *Ibid.*, pp. 113, 167.

(103) TACQUET : « Opera Mathematica », 1669, fig. 13 after p. 106 ; also « Elements or Principles of Geometrie » 1684, fig. for thm. IX, p. 5.

(104) Capital and small letters were first so used by EULER in the 18th century ; this use of Greek letters only became common in the 19th century. TROPFKE : « Geschichte », v . IV, p. 15.

(105) « Reorganization of Mathematics », p. 78.

or, omitting the latter entirely, increase the number of axioms. ARDÜSER's text (106) has no postulates, but lists twelve axioms, the ninth and tenth of which are, respectively, the fourth and fifth postulates of EUCLID. Likewise, HOFMANN (107) has no postulates, and even gives the axioms along with the general definitions, numbered continuously with them. « The Elements or Principles of Geometrie » (108) gives fourteen axioms, the last five of which are called axioms « of proportion ». But near the end of the book, at the beginning of a list of « problems », he makes the suppositions that a right line may be produced in any direction, or may be drawn between any two points, and that a circle may be described about any point as center and with any radius. Among the authors whose works contain no mention of either axioms or postulates are METIUS (109) and GRUBER (110).

The 1685 edition of LAMY makes a distinction between axioms and postulates or « demandes », and gives a list of eleven of the former. But he introduces the latter only wherever they are needed. He defines his postulate, « supposition ou demande », as a proposition that is not as evident as an axiom but clear enough so that the truth of it will not be questioned (111).

Yet another idea is brought out in the very popular practical work of LE CLERC (112) where axioms and theorems requiring proof are classed together under the title of « Notions ». On the first page after the title page he defines this term as principles which are either self-evident or can be known by incontestable demonstration (113).

Among the few works which differ widely from EUCLID and at the same time preserve the distinction between axioms and postulates is found the « Cursus Mathematicus », by WILLIAM LEY-

(106) « Geometriæ, Theoricæ et Practicæ », 1627.

(107) « Teutscher Euclides », 1653.

(108) Published in 1684.

(109) « Manuale », 1646.

(110) Second ed., 1702.

(111) P. 1 after pref. « C'est une proposition qui n'est pas si évidente qu'un Axiome, mais aussi qu'on ne peut contester ; ainsi on demande qu'on l'accorde, pour n'être pas obligé de la démontrer. »

(112) « Traité de Géométrie », 1690.

(113) « Ce Traité de Géométrie est divisé en dix Chapitres. - - - Le II. établit des principes que j'appelle Notions, & qui sont des vérités évidemment connues par elles-mêmes ou par des démonstrations incontestables ».

BOURN (114). On page 221 he gives the « postulates, or grantable requests », six in number ; and on the four pages following this is his list of then axioms. He gives an explanation to each principle, and usually illustrates with a figure or a numerical case.

It is found that in almost all of the works of this period, including those very similar to EUCLID, that the number of axioms and postulates, if given at all, exceeds the number of five (115) as given by EUCLID. Sometimes but three postulates are given, as, for example, in the book of GEORGIUS HILARIUS (116). The fourth and fifth of EUCLID's postulates are included among the axioms, of which HILARIUS has thirteen. This is the usual thing when the postulates are given but in less number than five. In several of the works, as in Leybourn's « Cursus Mathematicus » (117), one finds as a postulate : « Two right Lines include not a Superficies. » This was introduced after the time of EUCLID, but is considerably earlier than our period. (118) The one chief source, however, of the extra number of axioms in the works of the seventeenth century is found in the expansion of the general axiom now often given : « If the same operation be performed upon equals, the results will be equal». Thus. by applying the operations of addition subtraction, multiplication and division (except, of course, by zero), squaring, extracting roots etc.. a number of axioms can be obtained. This took place rather slowly, however, the extension to roots and powers not being reached until 1798 (119). The last generalization in multiplication was reached by BORELLI in 1658 (120), while in ROHAULT's « Oeuvres Posthumes » is found the general axiom on division (121).

Theory of Parallels and the Parallel Postulate.

Three different definitions are given of parallels by the writers

(114) « Cursus Mathematicus », 1690.

(115) According to T. L. HEATH : « The Thirteen Books of EUCLID's Elements », v. I, p. 154. Hereafter referred to as HEATH : « EUCLID ».

(116) Progymnasmatum Mathematicorum Enchiridion », 1656.

(117) Edition of 1690, p. 221.

(118) HEATH : « EUCLID », v. I, p. 232.

(119) TROPFKE : « Geschichte », v. IV, p. 31.

(120) *Ibid.*, v. IV, p. 31.

(121) Ed. of 1682, p. 10, axiom 7 : « Les grandeurs qui sont moitié, ou tiers, ou quart &c. d'une même Grandeur ou de Grandeurs égales, sont égales entr'elles. » See also TROPFKE : « Geschichte », v. IV, p. 31.

of the leading texts of the period : 1. Those which bring out the thought that parallel lines do not meet, 2. Those which emphasize the idea of inclination or direction, and 3. Those defining parallels as lines everywhere equidistant from each other. The definition most frequently used is the first, which, as is well known, is the Euclidean form. The greater proportion of the texts present this definition, usually giving also one or both of the other explanations. More than half of the number include in the definition the statement that parallels are everywhere the same distance apart. Among those are three of the most widely used texts of the seventeenth century, namely, those of LAMY (122), LE CLERC (123) and PARDIES (124). This supports the theory that the third definition mentioned above was used about as much as any by the close of the century. This equidistance theory of parallels was not unknown to the ancients, for the idea occurs quite clearly in their writings, as, for example, in a quotation from GEMINUS given by PROCLUS (125). But not until the sixteenth century did it find its way into textbooks. TROPFKE mentions (126) RAMUS as the first to have introduced it, in his work of 1569 (127), but in « The Pathwaie to Knowledge », 1574 (128), ROBERT RECORDE gives it alone as the definition of parallels. He says, « Paralleles, or Gemowe (129) lines bee suche lines as bee drawen forthe still in one distaunce, and are no nerer in one plate, then in an other, for an if thei bee nerer at one eande then at the other, then are thei no paralleles ». (130).

The idea of direction or inclination is far less common. Some works define two lines as parallel if when they are cut by a transversal, they make equal angles with it. « The Elements or Principles of Geometrie » (131) defines as follows : « Parallel Lines are those which lean not at all towards one another : So that if they were drawn out infinitely they wou'd never meet ». Some con-

(122) Op.cit., 1685 ed.
(123) « Traité de Géométrie », 1690.
(124) 1st. ed., 1671.
(125) HEATH « : EUCLID », v. I, p. 191.
(126) « Geschichte », v. IV, p. 54.
(127) P. 12 of 2nd. pagination.
(128) 1st. ed., 1551.
(129) From the French jumeaux, twins.
(130) *Ibid.*, p. 7 unnumb. of defs.
(131) Chap. I, def. 11.

ception of all three definitions can be detected in ARDÜSER's interesting way of putting it : « Lineae Parallelae gleich weitige oder gleichlauffende linien/ seyn die so sie neben ein andern zogen werden sich niemal schneiden oder zesammen lauffen/ ob sie gleich ohne endt fort zogen wurden » (132). Here, in the words « gleich weitige » we have the equidistance theory, in the adjective « gleichlauffende » is seen the direction theory, and in the remainder of his definition is expressed the Euclidean idea.

The problem of the parallel postulate, while an exceedingly live one (133) in this century, is nevertheless of little concern to us in a consideration of these books. The question of the need or possibility of demonstrating it does not arise here. But it is of value to observe what attention was given to it in the texts. This study shows that only about one-fourth of the works use it as a postulate or axiom, and then almost invariably in the same form as stated by EUCLID. In other books it is stated as a definition, sometimes with a few lines of explanation. Finally, in a considerable number of volumes it appears here and there as a tacit assumption rather than an explicit statement.

Triangles.

The common practice of seventeenth century writers, whether theoretical or practical, was to enlarge upon explanation and abbreviate demonstration. This resulted in the giving of a large share of space to definitions and the attempt at clarifying them. Even works like that of HERIGONE (134) which aims at brevity, neither shorten definitions nor reduce the number of them. Many books do more defining than EUCLID did.

This fact is illustrated in the geometry of the triangle. Almost any text contains a more elaborate classification of triangles than any writer of the present time would care to give (135). This is especially noticeable in works carried over from the late sixteenth century. BAROCIUS may be taken as an example. His work (136) is mostly descriptive and seems to lay great importance upon defining and illustrating the different kinds of triangles. But the

(132) Ed. of 1627, def. 37, f. 2.
(133) HEATH : « EUCLID », v. I, p. 210, on the attempt of JOHN WALLIS to prove it.
(134) « Cursus Mathematicus », 1634.
(135) « Reorganization of Mathematics », p. 76.
(136) « Cosmographia », 1585 and 1598.

usual procedure of all writers of this time is to give the definitions more, it seems, as a matter of form than for any other reason. The widely used work of LE CLERC (137) may be used as an example. On pages 5 and 6 the author defines plane rectilinear figures of all numbers of sides from three to twelve, after which he gives the definitions of « rectangle » (right triangle), « triangle ambligone » (obtuse triangle), « triangle oxigone » (acute triangle), equilateral, isosceles and scalene triangles. He illustrates the polygons with a pentagon, and each kind of triangle with a special figure. He gives more than a page to these definitions, but some of the terms are never used again throughout the work.

Some changes in the proof of propositions concerning triangles are noteworthy. In proving the sum of the three angles of a triangle equal to two right angles, EUCLID produced one side and divided the external angle thus formed by a line parallel to the side of the triangle opposite it. While that proof is still favored by most writers, LAMY (138) and PARDIES (139), two extremely popular authors, prefer the Pythagorean proof (140). LAMY draws through the vertex a line parallel to the base and shows the two angles thus formed equal to the angles at the base, by the equality of alternate-interior angles. This same proof is given in a scholium on page 104 of the 1607 edition of CLAVIUS. (141) PARDIES, proceeding in a slightly different way, draws the line from the vertex in only one direction, and uses both the equality of alternate-interior angles, and the converse of the parallel postulate.

Propositions relating to the four sets of concurrent lines (142) receive more attention in this period than was given them by EUCLID. Only the two relating to angle- and perpendicular-bisectors are treated in EUCLID's Elements although the others were known as early as the time of ARCHIMEDES. Here and there in seventeenth century works, proofs for the concurrency of the medians and altitudes of a triangle are also given. In the German edition of MAROLOIS (143) the former is given as follows : « In allen Triangeln/

(137) « Traité », 1690.
(138) Ed. of 1685, p. 63.
(139) Ed. of 1725, p. 14.
(140) TROPFKE : « Geschichte », v. IV, p. 68.
(141) « Euclidis Elementorum Libri XV ».
(142) Angle-bisectors, perpendicular- bisectors, medians and altitudes.
(143) « Opera Mathematica », 1638, thm. IV, p. 62.

wann die drey seiten in zwey gleiche theile zertheilet seindt/ und dass von den gegen obergesetzen Winckeln rechte Linien gezogen seindt/ so werden sich dieselben zugleich/ oder eins die andere in einem Puncto durchschneiden also dass der theil beim Winckel zweyfach mit dem Rest sein wird. » Then follows the «Auslegung». Theorem V states that the altitudes cut in a common point, and theorem VI deals with the same proposition when it is necessary to produce the altitudes to get them to intersect. Other works of this period concerned with these theorems are those of CLAVIUS (144), VAN CEULEN (145), ARNAULD (146) and HOFMANN (147). The last named author gives on page 67 a much more cumbersome proof than is found in modern texts.

Other Polygons.

Only a few changes in the propositions relating to the quadrilateral are effected in these years. The widest differences in this subject are found in the confusing terminology, already referred to above. The common tendency is to pass lightly over those propositions not mentioned by EUCLID. Some additions, however, are found in certain volumes. For example, in CLAVIUS' EUCLID (148) are found the previously missing theorems to the effect that a quadrilateral is a parallelogram if the opposite sides are equal (149) or if the opposite angles are equal (150).

Not much is done with the trapezoid. HOFMANN makes no distinction between the trapezoid and trapezium as they aren defined in the earlier modern texts. The trapezium may, according to him, have either two or no sides parallel (151). After having given the definition he does no more with it. METIUS satisfies himself with a special case or two. He divides (152) quadrilaterals---

(144) « Geometrica Practica », 1604.

(145) Ed. of 1615.

(146) « Nouveaux Elemens », 1690.

(147) « Teutscher Euclides », 1653.

(148) 1607, pp. 110, 111.

(149) « Omne quadrilaterum habens latera opposita æqualia est parallelogrammum ».

(150) « Omne quadrilaterum habens angulos oppositos æquales, est parallelogrammum ».

(151) Op. cit., 1653, p. 92. « Ein Trapezium ist eine Vierseitige Figur/ dessen Seiten nur zwo oder gantz nicht Parallel sind/ wie nachtfolgende vorzeigen ».

(152) « Manuale », 1646, p. 118.

« Viersydige » or « quadrangula » — into two groups, the « even-wijdighe », or Parallelogramma », and the « Niet evenwijdigh », or « Trapezia ». Of the latter there are two kinds, « rechthoekigh », and « slimhoekigh ». Of these, the former is defined as a quadrilateral having two sides parallel and a third side perpendicular to these parallels (153). To get the area he multiplies the half-sum of the parallels by the perpendicular. The latter are the general quadrilaterals, and he divides them into triangles in his treatment of their mensuration. SIMON STEVIN (154) deals in a practical way with the trapezoid. With the four sides given he shows how, by drawing a line through a vertex parallel to one of the non-parallel sides, a triangle is formed the three sides of which are known. The altitude of the triangle can be obtained, and hence the altitude of the given trapezoid. In general, use of the trapezoid in practical works is fairly common, but only scant treatment is given in the more theoretical books. Seventeenth century works are strangely silent on the now commonly used line joining the mid-points of the non-parallel sides.

Polygons with reflex or reentrant angles are given more attention than elementary geometries of today are wont to do (155). HOFMANN, in defining a trapezium illustrates with several figures, one of which contains a reentrant angle (156). On page 21 of his « Oeuvres » (157) SIMON STEVIN defines two kinds of quadrangles, according as they have a reflex angle without the sides cutting each other, or one with the sides cutting each other (158). METIUS shows numerous figures containing reflex angles (159), as do also DECHALES (160) and LE CLERC (161).

The well known propositions relating to the sum of the interior

(153) *Ib.*, p. 122. « By een rechthoekige Trapezium verstaen wy een figuir, van 4 syden beklemt, waervan de 2 overige syden zijn paralleel ende de eene syde is met den selven perpendiculaer ».

(154) « Les Œuvres Mathématiques », 1634, p. 373 2nd. pagination.

(155) Reorganization of Mathematics », p. 76.

(156) 1653, p. 92,

(157) 1634, 2nd. pagination.

(158) « Un quadrangle ayant quatre angles sans angle revers, & sans côtés coupant l'un l'autre, nous l'appellons quadrangle vulgaire : Mais ayant un angle revers ; quadrangle d'angle revers ; Et quand il a deux côtés coupant l' un l'autre quadrangle croisé ».

(159) « Manuale », 1646.

(160) « Mundus Mathematicus » , 1690.

(161) « Traité », 1690

angles and exterior angles of a polygon (162), although absent from EUCLID's text, were known long before the period covered by this investigation, (163) but not until this period did they win wide acceptance. A considerable number of seventeenth century works include them. (164) In the anonymous « Elements » (165) the author announces the theorem, « All the external Angles of any Polygon are equal 2 right Angles», and in the discussionof this theorem he takes any point within the polygon and joins it with each vertex. Then he shows that the polygon has « twice as many right Angles, (as it has sides;) except4, which are about the point in the middle ». The same method is used by PARDIES (166), to which he adds the remark that the same conclusion can be arrived at by dividing the polygon by means of diagonals — as is done at the present time. LAMY treats the theorem concerning the sum of the internal angles of a polygon (167). Both propositions are dealt with by ARDÜSER as corollaries 2 and 3 under the theorem concerning the sum of the three angles of a triangle(168). HOFMANN (169) combines both propositions into one, as follows : In any polygon the sum of the internal angles is equal to twice as many right angles as the polygon has triangles in it ; but the external angles together make only four right angles (170). PETER RAMUS also considers these propositions (171).

The Circle.

Many of the propositions relating to the circle found in texts of this period but not in EUCLID's are additions to, or converses of, EUCLID's theorems. Some of these were no doubt recognized by EUCLID but were not incorporated in his work, as he frequently

(162) Namely, the sum of the interior angles of an n-gon = 2 (n-2) right angles, and the sum of the exterior angles of any polygon = 4 right angles ».

(163) TROPFKE : « Geschichte », v. IV, p, 102.

(164) More, it seems, than suggested by TROPFKE : « Geschichte », v. IV, p. 102.

(165) 1684, p. 74.

(166) 1725, p. 25.

167) 1685, p. 84.

(168) 1627, f. 10.

(169) 1653, p. 69.

(170) « In einem jedem Triangulat gleichen sich die innern Winckel zusamen mit noch so viel rechten Winckeln/ als es Triangul in sich hat ; die eussern aber zusamen thun nur vier rechte ».

(171) 1599, pp. 45, 46.

left obvious material to the reader. The 1607 edition of CLAVIUS has a large number of good examples of such theorems. Several of the propositions relating to the measurement of angles are completed in his work (172). Also many scholia are given in which alternate proofs are offered and other ideas presented. This work also gives numerous points of view of other authors. PROCLUS, COMMANDINUS, FINAEUS and others are mentioned (173). PETER RAMUS likewise makes a number of changes and additions. (174) The method now used of drawing a tangent from a given external point to a given circle, by constructing a semicircle upon the line joining the given point and the center of the circle as a diameter (175) makes its appearance in some of these textbooks. The Euclidean method (176) still prevails, however. HOFMANN's book (177) is one of the few which contains only the modern method, while RECORDE (178), HERIGONE (179), RAMUS (180), ARDÜSER (181,) CLAVIUS (182) and ROHAULT (183) still use the auxiliary circle concentric with the given one. LAMY (184) gives both methods.

The proposition now commonly given concerning the equality of the two tangents from a given point to a circle, while not mentioned in EUCLID, is quite common in the works of this period. RAMUS, CLAVIUS, PARDIES, and the writer of the « Elements or Principles of Geometrie » are some of the authors who give it. CLAVIUS mentions it as a corollary on page 310 of the 1607 edition (185). The « Elements » gives it as a theorem (186).

It is rather unusual to find any reference to the common tangents

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(172) 1607, p. 279.

(173) *Ibid.*, pp. 62, 138, 182, 240, 266.

(174) Note ed. of 1599.

(175) First used, according to TROPFKE's « Geschichte », v. IV, p. 116, by VOEGELIN in 1549.

(176) HEATH : « EUCLID », v. II, p. 43.

(177) 1653, p. 107.

(178) 1574, bk. I, xxiiij. concl.

(179) 1634, p. 125.

(180) 1599, p. 109, 2nd. pag.

(181) 1627, f. 28.

(182) 1607, p. 267.

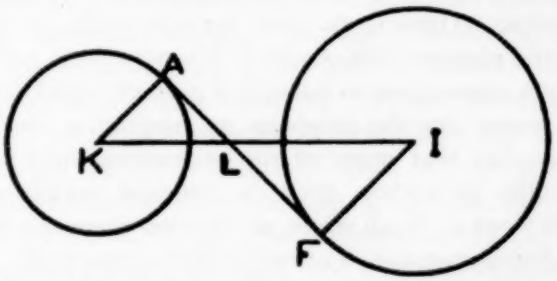
(183) 1682, p. 133.

(184) 1685, pp. 39, 40.

(185) « Constat etiam, duas rectas ab eodem puncto ductas, quæ circulum tangant, inter se esse æquales ».

(186) 1684, p. 45.

to two given circles, now so universally included in a plane geometry course. LEYBOURN (187), ROHAULT (188), PARDIES (189), LAMY (190) and HOFMANN (191) omit these propositions. The 1607 edition of CLAVIUS (192) treats the common external tangents quite fully, but it is left to GIOVANNI CEVA to take up the case of internal tangents. His treatment is as follows : « Sint deinde, vt in duobus reliquis casibus, circuli inaequales, & producta KI in tertia figura conueniat in L cum contingente AF pariter producta ad partes circuli minoris, quemadmodum in eodem puncto L contingens in secunda figura occurrit eidem KI. Quoniam utraque upsarum KA, IF perpendicularis est ad AF, erunt inter se aequidistantes, and propterea ut KA ad IF, ita KL ad LI ; componendo autem in secunda figura, & in tertia diuidendo, erit KI ad IL, vt cõpositum ex duabus KA, FI in secunda figura, sed vt earum



differentia in tertiâ, ad eandem HI. Itaque cum tam ratio compositi, quam differentiae duorum praedictorum radiorum ad radium minorem data sit ; & item data sit longitudine, ac positione antecedens KI, dabitur quoque consequens IL, & punctum L ; quare cum ab eodem puncto L ducere possimus vnicam tantum lineam contingentem IF, & vnicam alteram tantentem circulum KA, necesse est vt istae cadant in contingentes FL, LA, seu AFL, quam à principio posuimus tangentem duos circulos. Compositio problematis manifesta est. » (193)

(187) « Cursus Mathematicus », 1690.
(188) Ed. of 1682.
(189) Ed. of 1725.
(190) Ed. of 1685.
(191) Ed. of 1653.
(192) P. 268.
(193) 1678, p. 52, fig. 60.

Proportion and the Similarity of Polygons.

The subject of proportion is given generous treatment by the geometers of this period. Doubtless this is to some extent due to the fact that the various branches of seventeenth century mathematics were not divided by such sharply drawn lines as was the case at the beginning of the present century. Some attempt was made to show the correlation between arithmetic and geometry, and between algebra and geometry. For example, the « Cursus Mathematicus » of WILLIAM LEYBOURN (194), in treating the second book of EUCLID, handles it as a « parallel between arithmetick and geometry » and gives both geometrical and arithmetical demonstrations. ARDÜSER's work (195) devotes much space to the processes of arithmetic and the manner in which they bear relationship to geometric procedure. WILLIAM OUGHTRED's « Clavis Mathematicae » (196) treats most subjects taken up both algebraically and geometrically. ANDREAS TACQUET (197), and similarly many others, gives a general mixture of geometry and trigonometry. Another reason for the emphasis on proportion seems to be due to the fact that many of the geometrical instruments and much of the calculation done in practical mathematics were dependent upon it. Such works as « Pratica vniversale facilissima, et breue di misurare con la uista » of MALOMBRA (198) containing a treatment of the methods of finding inaccessible heights, depths and the like, are almost wholly dependent upon the theory of proportion. We find that WILLIAM LEYBOURN gives 10 pages, folio size, to definitions and the discussion of this subject ; « The Elements » (199), a book of only 141 pages, begins the theory of proportion on page 52 and uses it throughout the remainder of the book ; and BLONDEL (200), TACQUET (201) and STEVIN (202), while passing rather lightly over the theory all make wide application of the subject.

To the theorem stating that a straight line parallel to one side

(194) 1690, p. 243.
(195) Ed. of 1627, Book II.
(196) Ed. of 1652.
(197) « Opera Mathematica », 1669.
(198) Ed. of 1630.
(199) Two eds. in 1684.
(200) « Cours de Mathématique », 1699.
(201) « Opera », 1669.

of a triangle divides the other sides proportionally, RAMUS adds the generalization found in modern textbooks, that if two straight lines are cut by a number of parallel straight lines, the corresponding segments are proportional (203).

It is again CLAVIUS who adds to the last case of the proposition relating to the product of the segments of two intersecting chords, making it include also the condition that the chords may intersect without the circle, i.e., the case of two secants through a fixed point outside the circle. He did not, however, make any attempt to state the theorem in a general form (204).

The theorem of PTOLEMY to the effect that the product of the diagonals of any inscribed quadrilateral equals the sum of the two products obtained by multiplying together pairs of opposite sides, is now no longer included in elementary geometries. It is given occasionally in seventeenth century works. LAMY includes it, and adds the remark that it is considered important by geometers on account of its value in the construction of tables of sines (205). PTOLEMY's tables of chords having been of great importance until the fifteenth century, it is to be expected that this proposition would still find favor in the texts. Indeed, perhaps because it is in itself an interesting proposition, it is still included in some books towards the beginning of the present century (206). ARDÜSER states it without giving valid proof and, immediately after it, takes up the problem of constructing, out of four given unequal lines, a quadrilateral about which a circle can be circumscribed (207). The 1652 edition of OUGHTRED's « Clavis Mathematicae » (208) also includes it.

Regular Inscribed and Circumscribed Polygons.

The « Golden Section », or the proposition relating to the

(202) « Œuvres », 1634.

(203) 1599, p. 43, 2nd. pagination. « Si lineae rectae parallelis pluribus rectis intersecantur, intersegmenta sunt proportionalia & contra ».

(204) 1607, pp. 310, 312.

(205) 1685, p. 275. « Cette proposition est très estimée par les Géomètres, à cause de l'usage étendu qu'on en peut faire dans la construction des tables des sinus».

(206) BEMAN and SMITH : « New Plane and Solid Geometry », 1899, p. 228.

(207) 1627, p. 187. « Von vier ungleichen graden linîen/ ein viereck zu schreiben/ darumb ein Circkel mag geschriben werdcn/ wie es MAROLOIS in seiner Geometria beschreibt. »

(208) P. 70.

division of a line in extreme and mean ratio,occurs twice in EUCLID's Elements (209) and was first constructed, as given in modern texts, by HERON OF ALEXANDRIA in the first century (210). But in seventeenth century works no particular attention is accorded it. Besides the editions of EUCLID, the « Geometrie » of HOFMANN (211) gives the same construction as is found in the « Elements ». (212) Among the few works that use the method of HERON are those of OUGHTRED (213) and LAMY (214). ROBERT RECORDE's method (215) is pure guess work. It is interesting to note that he seems to feel the need of an isosceles triangle with the angle at the base equal to twice the angle at the vertex, just as was done in the correct construction of EUCLID, before he proceeds to approximate his regular inscribed pentagon. This does not mean, however, that the proposition was unknown to all those writers whose works treat it lightly, but it seems rather that it was one of those to suffer neglect because of the transition from rigor to practice in elementary works. Mathematicians of the time who devoted themselves to research gave much consideration to it. RAMUS discusses it in « Scholarum Mathematicarum » (216) and sees in it an analogy to the divine Trinity. KEPLER was also interested in it. But the reference to it as the « Golden Section » was not made until two centuries later (217).

Turning to a number of works we find, for examples, that ALSTED (1641), TAYLOR (1707), METIUS (1646), DE GRAAF (1706) and the anonymous « Elements or Principles of Geometrie » (1684) have either nothing or very little material on these figures. RECORDE, already mentioned, LE CLERC (218), and ARDÜSER (1627) use approximation methods. RECORDE's simple instructions are to divide the circle into as many equal arcs as the polygon to be inscribed has sides. After giving this direction for inscribing a polygon of fifteen sides, he adds : « And so doe with any other

(209) Bk. II, 11 and Bk. VI, 30.
(210) TROPFKE : « Geschichte », v. IV, p. 185.
(211) 1653, pp. 89, 90.
(212) HEATH : « EUCLID », v. II, p. 267.
(213) 1652, p. 75.
(214) 1685, p. 119.
(215) 1574, XXXVII. and XXXVIII. conclusions
(216) 1627, p. 191.
(217) TROPFKE : « Geschichte », v. IV, p. 187.
(218) Ed. of 1690. See below for his accurate method.

figure, of what number of sides so euev it bee. » (219) LE CLERC simply states in « Traité de Géometrie » that, to inscribe a regular pentagon one must divide the total number of degrees in the circle by 5 and the quotient will be the measure of the arc required (220). He also gives approximation methods for the seven and nine sided regular polygons.

In the « Nova Geometria » (221) LE CLERC gives an approximate construction for the regular pentagon upon a given right line as side. But when the circle is given and it is required to inscribe a regular pentagon, he gives further on (222) the exact and very convenient method which is due to Ptolemy of the second century. No proof is offered. The reader will readily follow his directions without a translation :

« Praxis.

Ducantur duae diametri	AB, CD.
angulis se rectis intersecantes in	E.
Semidiameter	CE.
bifariàm secetur in	F. (mid-pt. of CE.)
Ab hoc puncto	F.
intervallo	FA.
describatur arcus	AG (G is on CD.)
A puncto	A.
intervallo	AG.
describatur arcus	GH.
Recta	AH.

erit unum Pentagoni latus. »

ARDÜSER uses the exact method for the regular pentagon, and then adds approximation methods for the regular figures of seven, nine and eleven sides (223). PARDIES' treatment is rather meager and slip-shod. In the construction of a regular pentagon upon a given line as side, HOFMANN proceeds in the following rather unique

(219) « The xlvj. conclusion ».

(220) 1690, p. 59. « Divisez le nombre des degrés du cercle entier par le nombre des côtés du polygone, c'est à dire, divisez 360 par 6, & le quotien 72, sera l'angle du centre --- que vous ferez pour avoir un arc dont la corde --- soit un des côtés du Pantagone demandé ».

(221) 1692, p. 82.

(222) Ibid., p. 112.

(223) 1627, ff. 173-175.

manner (224); 1. Let AB be the given side. 2. Divide AB in extreme and mean ratio. 3. With the sum of AB and its larger segment so divided, as radius, and A and B as centers, strike two arcs, intersecting in C. 4. With AB as radius and A and B as centers, strike arcs cutting the previously made arcs in D and E. 5. ABECD is the required pentagon. The reader will readily see the reasoning employed in the above procedure.

Other propositions were introduced here and there in the elementary texts as they were brought to light through the efforts of investigators in more advanced fields. An example of this in connection with the regular polygons is found in ARDÜSER's book (225) where the author states the theorem that the area of a regular inscribed polygon is the mean proportional between the areas of the regular polygons of half the number of sides inscribed in and circumscribed about the same circle. (226) He takes the special case of the hexagon for his proof and then states in a corollary without proof that the same relation would obviously hold for the general polygon. That the perimeter of an inscribed regular polygon is the mean proportional between the perimeters of the circumscribed regular polygon of the same number of sides and the regular inscribed polygon of half the number of sides was first stated by SNELLIUS in 1621 and proved by HUYGENS in 1654 (227). Our older modern textbooks in geometry sometimes include these proportions. (228)

Transformation and Division of Areas.

One feature in which there exists a wide difference between the works being examined and present-day texts is in the emphasis upon theorems relating to the transformation and division of figures. In the « Practical Geometry » of RUDD (1650) there are two parts, the first of which is almost entirely on the manipulations of areas ; VAN CEULEN (1615) devotes 53 of his 271 pages

(224) 1653, pp. 94, 95.

(225) 1627, ff. 185, 186.

(226) « Die Regular rechtlinischen Figuren zwischen den Regular rechtlinischen Figuren| so halbs weniger seiten haben/ und inn unnd umb den Circkel geschriben werden ».

(227) TROPFKE : « Geschichte », v. IV, p. 192.

(228) BEMAN and SMITH : « New Plane and Solid Geometry », 1899, p. 219.

to the changing of figures ; much of the fifth and sixth books of ARDÜSER's work (1627) is on the same subject ; chapters IV and V of LE CLERC's « Traité » (1690) are of similar content ; and similarly in a large number of seventeenth century texts.

The reason for the great stress on the application of areas is accounted for largely by the undeveloped state of algebra at that time. For many of the propositions connected with this subject provide a solution by geometry for problems now taken care of in algebra. For example, the twenty-eight proposition of EUCLID's Elements, VI, « is the geometrical solution of the quadratic equation $ax - \frac{b}{c} x^2 = s$, subject to the condition necessary to admit of a real solution ». (229) Under those circumstances the theorems on geometrical algebra were entirely in place. But with our modern courses in algebra occupying the position they now do in the curriculum, to introduce into geometry these theorems on the application of areas, interesting as they are, would be to require the pupil to spend much time in unnecessary duplication of material.

Another reason for the stress on the application of areas is found in the prominence given the subject by EUCLID. The tendency of seventeenth century writers was to combine into one subject of study practical geometry and EUCLID's Elements, which in the sixteenth century were looked upon as distinct courses. It is only natural that material on the application of areas should find its way into their productions.

However, much of the material brought in in connection with this subject, especially that pertaining to the divisions of figures, is also found in the construction work of modern texts.

The Pythagorean Theorem and the Lunes of Hippocrates.

The famous theorem of PYTHAGORAS received due recognition. All practical works made use of it and the theoretical books demonstrated it. Several of the numerous (230) proofs for this proposition are seen in these writings. A goodly number of authors use the proof usually seen today, and due, according to PROCLUS, to EUCLID. That is to say, they divide the large square into two rectangles,

(229) HEATH : « EUCLID », v. II, p. 263.
(230) TROPFKE : « Geschichte », v. IV, p. 141. footnote

form triangles by joining the vertex of the right angle with the two distant corners of the large square, and by connecting each other vertex of the triangle with the far corner of the square on the opposite side, and arrive at the proof by means of the congruency of certain of these triangles. This is the method used among others by LAMY (231). HOFMANN arrives at the proof through the following propositions (232): 1. The perpendicular dropped from the vertex of the right angle to the hypotenuse forms two triangles similar to each other and to the given one ; and is itself the mean proportional between the segments into which it divides the hypotenuse. Moreover each of the other sides is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that side. 2. If similar triangles be constructed upon the sides of a right triangle, the sum of the two on the legs is equal to the one on the hypotenuse. Then, by properly drawing the diagonals of the three squares, he gets three similar triangles and applies proposition 2 above. Doubling each term of the equation brings the desired result. While PARDIES brings in no new principle, his proof is nevertheless attractive (233): Let ABC be the given right triangle and BCMN the square on the hypotenuse. Draw AE perpendicular to BC, cutting BC in D and MN in E. Now by a theorem already referred to (1 above), he establishes the proportion, BD : AB = AB: BC, or BN. Therefore, the square on AB = rectangle BE. Likewise the square on AC = the rectangle DM, and the proof is established.

A few of the works generalize the Pythagorean theorem and show that any polygon erected upon the hypotenuse of a right triangle is equal to the sum of two similar polygons erected upon the two legs of the triangle. This generalization is found in « De Arithmetische en Geometrische fondamenten », by VAN CEULEN (234), PARDIES' text (235), the anonymous « Elements or Principles of Geometrie » (236), and LE CLERC's « Traité » (237).

(231) 1686, p. 92.
(232) 1653, pp. 61, 62, 78.
(233) 1725, p. 67, 1st. ed. 1671.
(234) 1615, p. 110.
(235) 1725, p. 68.
(236) 1684, p. 88.
(237) 1690, p. 44. « Si on décrit des poligones semblables sur les côtés d'un triangle rectangle le plus grand, c'est à dire, celui qui aura pour base le côté opposé à l'angle droit sera égal aux deux autres ».

Since circles are to each other as the squares of their diameters, VAN CEULEN, PARDIES and the author of the « Elements or Principles of Geometrie » stated further (238) that the area of the semicircle on the hypotenuse as diameter is equal to the sum of the areas of the two semicircles on the legs as diameters. If the semicircle on the hypotenuse is placed on the same side with the given triangle, it will, obviously, pass through the vertex of the right angle. With the semicircles on the legs drawn outside the given triangle, there will be a part of each small semicircle common with the large one. Leaving out the common part, we have the given right triangle equal in area to the sum of the two lunes. In the special case of an isosceles right triangle, the two lunes are equal, and hence each is equal to half the triangle. This was the case considered by HIPPOCRATES in his efforts at squaring the circle.

Incommensurables and the Computation of Pi.

Not many of these works deal at great length with the subject of incommensurables. Occasionally one finds a text which gives the topic enough consideration to warrant the mention of it in this connection. The « Element Geometriae » of MERCATOR, 1678 edition, of nine books, devotes the seventh solely to a discussion of this subject. Likewise the seventh chapter of PARDIES' volume (239) is of similar content. Another work, and one that went through many editions, the « Eléments de Géometrie », by LAMY (1685), gives the fourth section of book three to theorems on the commensurability and incommensurability of lines and surfaces.

Seventeen theorems and the suggestion of the solution of some related problems are given by LAMY. The following will serve to illustrate the nature of his work (240) :

Theorem three. « Si les quarrez de deux lignes ne sont pas entr'eux comme deux nombres quarrez, ces deux lignes ne sont pas commensurables ». (If the squares of two lines are not in the ratio of two square numbers, they are incommensurable.) (241)

(238) 1615, p. 110 ; 1725, p. 68 ; and 1684, p. 89 respectively.
(239) Ed. of 1725 ; 1st . ed. 1671.
(240) 1685, pp. 155, 162.
(241) EUCLID's Elements, Bk. X, prop. IX.

Theorem fifteen. « Les deux parties d'une ligne rationelle coupée en moyenne et extrême raison, ne sont pas rationnelles. » (The two parts of a rational line cut in mean and extreme ratio are irrational.) (242).

In an « avertissement », however, at the beginning of the section he states that a prerequisite to the reading of this subject is the sixth book of the « Grandeur », an earlier publication of his, and adds that if one has not read that work, one may omit this section and that which relates to this topic in the remaining part of the work (243). It is clear that the author of this widely used text regarded this topic as one of the less desirable ones for study in elementary geometry.

The problem of the quadrature of the circle was still a subject of deep concern to mathematicians of the sixteenth century. ORONTIUS FINAEUS, first professor of mathematics in the University of Paris, and a mathematician of more pretense than performance, made great claims as to his success in the problem (244). The book of JEAN BORREL is one of the most scholarly works along this line. Besides describing various attempts that had been made up to his time, he refutes flatly the arguments of FINAEUS (245).

A considerable number of writers of the following century found it desirable to say something in their texts on the squaring of the circle, or, at least to give some approximate methods for changing circles into rectilinear figures of the same areas. The fifth book of ARDÜSER's text is given largely to such constructions. DE CHALES devoted the seven propositions of the ninth book of his « Mundus Mathematicus » to « De quadratura circuli ». In a note after the theorem stating that a circle is equal to a triangle whose base is equal to the circumference of the circle and whose altitude equals its radius, « The Elements or Principles of Geometrie » (246) has the following explanation : « And this is call'd

(242) *Ib.*, Bk. XIII, prop. VI.

(243) 1685, p. 152. « Cette Section suppose qu'on a vû le 6 livre de la Grandeur. Si on ne la pas lû, on peut passer cette section, & tout ce qui se dira dans le Livre suivant de l'incommensurabilité ».

(244) TROPFKE : « Geschichte », v. IV, p. 214. « Bald erkannte er die Annäherung in $\pi = 3\ 1/7$ an, bald rhümte er sich ‚mit einer Konstruktion, die in Wirklichkeit ein noch viel ungenaueres Resultat liefert, das grosse Problem endgültig gelöst zu haben ».

(245) 1554 and 1559, pp.42-50.

(246) 1684, p. 76.

the Squaring of a Circle, viz, to find a Rightlined figure equal to a Circle ; upon this supposition, that the basis given is equal to the circumference of the Circle. But actually to find a Rightline = to the circumf. of a o, is not yet discover'd Geometrically. » The work of WELLS (247) disposes of the difficulty in essentially the same way.

Beginning with the late sixteenth century (248) and extending throughout the seventeenth, much was done by mathematicians in the determination of more exact values of pi. But the textbooks of elementary geometry do not treat this subject very extensively. The method of calculation found in modern geometries was brought to its present form by HUYGENS, but was not generally given a place in geometry texts until the first quarter of the eighteenth century.

LUDOLPH VAN CEULEN, who gave much attention to the calculation of pi, uses in his work (249) the value 3.14159265. TAYLORS' « Treasury of the Mathematics » (250) uses the two values 22/7 and 355/113. METIUS discusses it more fully than the usual text of this time. He mentions (251) the fact that no person has determined this ratio exactly ; that the fraction 22/7 given by ARCHIMEDES is too large and that 3 10/71 is a bit too small ; and he gives his father (252) credit for the discovery of 355/113, which is both very nearly accurate and a convenient figure to remember. However, we are reasonably certain that this value was used by the Chinese as early as the fifth century A.D. (253).

Mensuration. Practical Geometry.

One of the very striking features of seventeenth century geometry is the wide popularity of practical works as shown by the large

(247) 1723, p. 263.

(248) VIETA : « Variorum de rebus mathematicis liber VIII », 1593.

(249) 1615, p. 110.

(250) 1707, p. 61.

(251) 1646, pp. 150, 151.

(252) «By mijn salighe Vader gevonden mikkelraminge, seggende als de Diameter is 113, woo geeft den omring des Circuls 355 gedeelten, is geen verschil van een 1/100,000 gedeelte, daerom dese proportie, sonder eenige merckelijke faute in een Circul, zijnde soo groot als de Circumferentie van 't aertrijck gebruyckt can werden. »

(253) YOSHIO MIKAMI : « The Development of Mathematics in China and Japan », Leipzig, 1913, p. 50.

number of texts which appeared to meet that demand. The number of books of this character still left to us is so great that no attempt can be made to name all of them. A few of these will suffice to illustrate. The Italian work of CAPRA (1673) describes the use of geometry in brick and stone work, and in the mensuration of areas, solids and casks. Then there is the « Cours de Mathematique » by BOURDIN which divides the subject into « speculative », « practical », « resolutive », « effective », « respective », and « military » geometry. The « speculative » geometry considers the nature of figures ; the « practical » is what we commonly think of as construction work ; the « resolutive » deals with the solution of the plane triangle ; the « effective » applies this « resolutive » geometry to the measurement of inaccessible altitudes, distances and the like ; the « respective » is concerned with the construction of figures similar to others and in given ratio to them ; and « military » geometry is the science of fortifications and warfare. (254). It is clear from the above that the entire work is concerned wholly with the practical applications of geometry.

Another distinctive work of this type is the German « Lust-Garten » of TOBIAS BEUTEL. The first book is composed of a treatment of plane figures, their construction, transformation, addition, subtraction, division, multiplication, and various measurements, including simple trigonometric calculations. The second book treats of the construction, transformation and measurement of solids. The last part contains about two hundred solved problems and constructions of geometry and arithmetic. This work was very widely used the last half of the seventeenth century.

Some writers distinguish two separate courses of study, the theoretical geometry of EUCLID's Elements and practical geometry. The posthumous work of ROHAULT (255) is illustrative of this. But while such an arrangement is not uncommon, the majority of writers find it most desirable to combine theory and practice in the right proportion to meet the needs of the users of the book. The tendency is to make that choice of material which will best arm the student for his future work as a mechanic, surveyor, seaman or soldier.

Among the foremost men of his time in this field was SEBASTIEN

(254) Ed. of 1661, p. 4.
(255) 1682, pp. 1, 335.

LE CLERC. It therefore seems fitting to give a little more than a passing glance at his texts. The one, « Pratique de la Geometrie », in the Latin edition of 1692, takes up successively the discussion of geometry in general, its origin, utility, the principles of geometry, definitions and explanations with splendid drawings to illustrate, and finally the axioms and postulates, all given with remarkable clarity. Then begins the first of the five books. The first is on preliminary constructions and the use of ruler and compass; the second deals with the construction of plane figures ; the third and fourth take up, respectively, the inscribing and circumscribing of polygons; and the last book is concerned with proportion. (256)

The other work of LE CLERC is the French edition of 1690, called « Traité de Geometrie ». This volume is divided into ten chapters, containing, briefly, definitions, necessary theory, preliminary practice with lines and simple constructions, the transformation of figures, their division, measurement, simple trigonometry, solids, with particular emphasis upon their mensuration, and, finally, the measuring of altitudes, depths and plots of land. He gives the contents as follows : « Ce Traité de Geometrie est divisé en dix Chapitres. Le I. contient les Définitions. Le II. établit des principes que j'appelle Notions, & qui sont des veritez évidemment connües par elles-mêmes, ou par des démonstrations incontestables. Le III. donne la pratique des Lignes & des Angles, & fait décrire les figures des Plans. Le IV. enseigne à transfigurer ces mêmes Plans, c'est à dire, à leur donner de nouvelles figures, sans en diminuer ou augmenter le contenu. Le V. apprend à les diviser. Le VI. montre comment il les faut assembler, et comment ou peut les augmenter ou diminuer de grandeur, selon quelque quantité proposée. Le VII. enseigne à les mesurer. Le VIII. contient la Trigonometrie ou la doctrine des Triangles par le calcul. Le IX. traité des Solides, & particulièrement de leur Toisé. Le X. enfin, donne la pratique pour le Terrain, où l'on voit comme on leve les Plans, comme on les trace, & comme on mesure les dimentions inaccessibles. »

The contents of the above two works may well be taken as highly illustrative of what was considered as worth-while subject-

(256) See index : « Liber I. de Linearum ductu. Liber Secundus de figurarum planarum circumscriptione. Liber 3. de figurarum inscriptione. Liber 4. de fig. circumscriptione. Liber 5. de lineis proportional ».

matter for geometry texts by the educators of the last half of the seventeenth century and the most of the eighteenth. For these works were edited and translated many times. Coming, as they did, from an author who was both a teacher and a designer — for LE CLERC was professor « à l'école des Gobelins » — they met the needs both of the classroom and the shop. Consequently, for more than a century these works — and especially the « Pratique » (257) — continued to be widely used, appearing in translation in at least five languages.

Trigonometry.

The practice of introducing some trigonometry into earlier mathematics courses is now becoming more and more prevalent. It is not a new idea in European countries, where the elementary notions of trigonometry are nearly always made known to the pupil before he completes the ninth school year. (258). But it is, generally speaking, still an innovation in the schools of the United States. However, we have begun to see that the meaning of sine, cosine and tangent and their simpler properties as functions, together with their use, and the use of simpler tables of their values, in the solution of right triangles, should be taught in secondary schools. This is true by reason of the ease with which the less mature pupil may understand numerical trigonometry, its usefulness in many occupations, and the naturalness with which it can be introduced either in connection with the study of proportion in algebra or of similarity and proportion in geometry. (259)

With this present new emphasis on trigonometric ideas in mind, an examination of the geometries of the period of our study becomes exceedingly interesting. For we learn that the idea of including the simpler notions of trigonometry in works on geometry is by no means peculiar to our present age. A goodly number of texts used in the seventeenth century may be cited which do that very thing. For example, the mathematical work of JOHN TAYLOR (1707) treats the various subjects in the following

(257) The 6th ed. of « Bowles's Practical Guometry », a translation of this work was printed in 1780 .

(258) « Reorganization of Mathematics », p. 161.

(259) *Ibid.*, p. 25.

significant order : arithmetic, the use of logarithms, explanation of sines, tangents and secants, geometry, and then only trigonometry.

The work of GAUTRUCHE (1668), a Jesuit of great ability, who made remarkable progress in mathematics for his century, is worthy of mention in this connection. This book treats of the subject in two parts : « Elementale Geometriae » and « Geometria Practica ». In the second part, after dealing with the principle and practice of the measurement of heights and distances, he devotes chapter III to the use of trigonometry (260). Continuing, he gives in the next chapter a five place table of sines from one to ninety degrees. Following these he places the study of planimetry and stereometry.

GRUBER (261) includes also an explanatory chapter on trigonometry between the subjects of planimetry and stereometry. He gives very little of the actual substance of the subject, however, meaning apparently only to give to the student a general notion of the nature of the study. But very little trigonometry is given in the popular work of LAMY. He defines sine, not, however, as a ratio, but as half the chord subtended by an angle twice the size of the given angle (262). He also includes a few theorems involving the sine (263).

Chapters VIII of LE CLERC' « Traité » (264) gives a splendid treatment of numerical trigonometry. He begins by discussing the possibility of solving a triangle when three of the six parts are given ; leads then to the definition of sine, tangent, and secant, using a figure just like our own unit circle, but regarding the radius as 100,000 to avoid the decimal point in the tables ; and concludes his introductory remark with an explanation of the tables. Following this the different cases (265) in the solution of the triangle are taken up, beginning with the simplest and leading to the more difficult ones. The author concludes the chapter with a very brief illustration of how logarithms can be used in these solutions.

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(260) 1668, p. 63. « De usu Trogonometri ».

(261) 1702, pp. 63-67.

(262) *Ibid.*, pp. 48, 49. « Sinus d'un arc, c'est la moitié de la corde du double de cet arc. Le sinus d'un angle, c'est le sinus de l'arc qui le mesure ».

(263) *Ibid.*, pp. 50, 51.

(264) 1690, p. 166.

(265) *Ibid.*, pp. 169-179.

Nineteen double pages of the geometry of ARDÜSER (266) contain his material on numerical trigonometry. He begins with definitions. Then the usual cases in the solution of the triangle are treated in a rather more complicated manner than is done by LE CLERC. The use of instruments is introduced in the latter part of the book.

Progressions, Logarithms and Conic Sections.

These three subjects are occasionally included in the texts. Arithmetical and geometrical progression and logarithms make up the matter discussed in the eight chapter of PARDIES' work. (267) But he treats them less fully and rigorously than is done by the average college algebra of today.

GRUBER (1702) introduces conic sections in a simple manner along with other plane figures. This is the custom adopted by a number of writers. Another example of this kind, although somewhat more formidable, is « La Pratique de Geometrie », by SIMON STEVIN (1634).

On the other hand, some books on geometry are intended to be of a more advanced nature than we are interested in this study. The one by DE CHALES (1690) may be mentioned to illustrate this type of work. From the beginning DE CHALES assumes that the reader is well acquainted with EUCLID's Elements. The following table will at once convey to the reader the contents of his book : « Liber Primus. Mensura linearum rectarum.

Liber II. Planometria seu dimensio superficierum.
Liber III. Stereometria, seu solidorum dimensio.
Liber IV. De circino proportionum.
Liber V. De lineis curvis.
Liber VI. De figuris circularibus & curvis superficiebus.
Liber VII. De soliditate corporum rotundorum.
Liber VIII. Nova methodus geometrica.
Liber IX. De quadraturâ circuli.
Liber X. De progressionibus geometricis.
Liber XI. De novo usu centri gravitatis. »

(266) 1627, ff. 194-213.
(267) 1685, p. 88.

Solid Geometry.

One of the best treatments of lines and planes in space is found in LAMY's « Elements » (1685). After a few definitions of the usual sort, the following postulations are made : « One may prolong, or conceive prolonged, a plane as far as necessary.

A right line cannot be in part on a plane and in part in the air.

Two right intersecting lines may be conceived to be in the same plane.

All of a triangle lies in one plane.

The intersection of two planes is a straight line.

If a right line AB cuts a plane in the point A, and if B is equidistant from all points of a circle described in the plane with A as center, then, and then only, is the right line perpendicular to the plane.

If a perpendicular to a plane is moved parallel to itself with its foot tracing a straight line in the plane, it will generate a plane perpendicular to the given plane.

If one plane intersects another, and if a line in the one is perpendicular to the other, then the two planes are perpendicular to each other (268).

Following these postulations is found a statement of sixteen theorems relating to intersecting lines and planes. The proof is given with each. Then comes a section on the generation and construction of solids, dealing with pyramids, cones, prisms, solid angles and the five regular solids. Section III is concerned with theorems on the surfaces of solids, and the development of those surfaces. Section IV gives in nineteen theorems a thorough treatment of congruence and equivalence in solids. The last section is on the five regular solids, how they are made and the manner of inscribing them in, and circumscribing them about, a sphere.

A total of seventy theorems besides a considerable number of solved problems are given in this text of LAMY.

The fifth book of the geometry of PARDIES (269) satisfies itself merely with the statement of most of the theorems on solid geometry. TAYLOR (1707) and LE CLERC (1690) have no theory at all. « The Elements or Principles of Geometrie » (1684) does not make a clear-cut distinction between plane and solid geometry.

(268) Pp. 166-169. These translations are nearly literal.

(269) 1725, p. 39.

Before the beginning of part two there is a chapter « of surfaces », containing fourteen theorems, all of which would be included in the opening chapter of solid geometry in modern texts. For example, these are some of the theorems given (270) : « Parallel Right-Lines are in the same plane. » « A Line that is Pp. (perpendicular) to two Lines crossing, is also Pp. to their Plane. » « Planes are Parallel that receive the same Pp. » Other propositions of the thirty-eight contained in this work on solid geometry are very similar to those found at the present time.

Theorems on spherical angles and polygons are usually omitted from these geometries. Knowledge of spherical triangles being of practical value chiefly in the study of astronomy, an introductory chapter to the study of astronomy or cosmography was often given, in which a treatment was given on spherical triangles and trigonometry. Thus SIMON STEVIN's « Cosmographie » is divided into three parts, the doctrine of triangles, geography and astronomy. The first of these is made up of four books on the construction of tables, plane triangles, spherical triangles, and problems dealing with the celestial sphere (271).

But in solid geometry, as in plane, the emphasis is placed by the majority of texts upon the practical. This can best be illustrated by giving a brief description of the geometry of WILLIAM LEYBOURN. The work has 129 pages, folio size, 72 pages of plane geometry, with 48 theorems and 84 problems, 24 of solid with 38 theorems and 18 problems, and 33 of mensuration. The problems are largely construction work, while actual calculation is taken up in the part on mensuration. In his own words, « The Method which I shall observe in the Mensuration of Geometrical Figures, Plains, and Solids, shall be threefold ; viz. I. By Decimal Arithmetick, of all the other ways the most exact and absolute. — II. By Artificial or Logarithmical Arithmetick. — III. By Logarithmical Scales and Compasses, as the most expeditious of any other. » (272).

The first section of the part on mensuration has to do with plane figures, the second treats of solid figures and the last deals with the « Variety of Dimensions, Linear, Superficial, and Solid,

(270) P.p 95, 97, 100.
(271) 1634, p. 1, 2nd. pagination.
(272) 1690, p. 301.

of the Five, Plain, Ordinate, or Regular Bodies, commonly called the Pythagorean or Platonic Bodies, Arithmetically, by way of Proportion, to a Millimillenary Part of the Integer, and analytically contracted. » (273) On page 299 he gives a table showing the various proportions of a sphere and the five regular solids inscribed in it. Taking the diameter of the sphere to be 2, he gives, to five decimal places, its circumference, the area of a great circle, the area of the surface and the volume of the sphere, together with the length of a side, area of the surface, and volume of each of the five regular solids inscribed in it.

The last of the three books of ERRARD's work (274) is on the measurement of solids. There are fourteen chapters in this book. In the seventh he treats the mensuration of the sphere, after which he devotes the remaining seven to the following unusual topics : 8. Cylinder with oval base, 9. Cone oval, 10. Spheroid, 11. Bodies with spiral bases, 12. Cone spiral, 13. Irregular bodies, and 14. « De la manière de peser ».

Exercises.

Writers on elementary geometry in the seventeenth century followed those of preceding centuries in the custom of including in their works only problems and exercises with complete solutions. Rarely does one find an exception. Among the writings including a great many exercises and problems may be mentioned the geometry of RUDD (1650) and the « Lust-Garten » of BEUTEL (1660).

The « Practical Geometry » of RUDD is in two parts, the second of which contains one hundred geometrical questions either with the solutions or directions for obtaining them. The following problems will suffice to show the nature of the work :

» There is a Triangle, as ABC, whereof the side AB is 13, and AC is 15, and BC is 14.

The Question is : How long is the Perpendicular Line AD ? »

« In this Circle is made a square of equall sides, as ABCD, (as great as possible) in such sort, that the segment ABF, or BCG is found to contain 224.

The Question is ; How much is the Diameter of the Circle ? »

« There is a Triangle whereof two sides are known, as AB 52

(273) *Ibid.*, p. 324.

(274) « La géométrie et pratique générale d'icelle », 1620.

and BC 56, about the same is circumscribed a Circle whose Dia meter is 65.

The Question is ; How much is the third side of this Triangle ? namely the side AC. » (275).

The last problem in the book « is left to the student ». (276) « And to conclude », he says, « I have put one Question which I leave unanswered, only for the practice of those who desire to recreate themselves in such like novelties. The Question is thus. A Gentleman hath a right angled piece of Land as ABCD, in lenght 640 foot, and in breadth 480 foot, this piece of Land he would convert into an Orchard and a Garden, but finding the same in the Winter to be anoyed with water, he would have the same raised one foot over the whole superficies. Not it is resolved to make a Ditch round about the same which shall be 6 foot deepe so that the Ditch may remain of equall breadth and equal depth, till there be so much earth come out of the same as will raise the whole superficies remaining within the Ditch one foot higher.

The Question is : How broad shall this Ditch be ? »

Of further interest in this direction is the widely used book of BEUTEL (277). The second part, the « Geometrische Gallerie », contains about two hundred questions and problems. In every case some help is given to obtain the solution, but quite often that help consists merely in a suggestion as to what principle or proposition should be used. Thus as a hint toward the solution of problem CLXX (278), which asks for the volume of a brewingvat of the form of a frustum of a cone, with upper and lower bases respectively 14 and 17 feet and with altitude 4 1/2 feet, the author refers to the first part of his text where the frustum is treated ; « Item, so zum Bier-Brauen in einem Brau-Hause ein grosser Braubottich solte ausgemessen werden, welcher die Form eines abgekürtzten Coni hätte, und unten am Boden im Diametro 17. Fuss, oben 14. Fuss, und in seiner Höhe 4 1/2. Fuss inwendig hielte, ist die Frage, wie viel Cubischer Fuss sein gantzer Inhalt seyn würde ? Antwort 852 Cubic-Fuss.

Wird allerdings nach dem Exempel der 7. Fürgabe des fünfften

(275) 1650, 1st., 57th and 66th questions.
(276) *Ibid.*, p. 139.
(277) Ed. of 1729, 1st. ed. 1660.
(278) *Ibid.*, p. 220.

Theils des 2. Buchs vorgehenden Lust-Gartens gemacht, und entspringt alsdenn daraus jetzt ermeldtes facit. »

Some of the problems relate to the five regular solids, which are copiously treated by BEUTEL, others on the sphere, and a great number pertain to military tactics.

In short, the text books of the seventeenth century are almost entirely lacking in original exercises as we now use them. But the few examples given above represent a considerable advance of these works over those of any previous century, and may be taken as indicative of the trend of textbooks toward present standards.

One other observation should be noted. Throughout the sixteenth and early seventeenth centuries it was customary to recognize two distinct branches of study, EUCLID's Elements and Practical Geometry. By the close of the century a happy combination of theory and practice had been reached that came nearer to our present ideals and standards than at any other time in the history of geometry. For shortly after this time the pendulum again began to swing toward the ultra-theoretical.

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René-François de Sluse et le problème des Tangentes

I. *Introduction.*

La figure de René-François de Sluse, chanoine de la cathédrale de Liége, est une des plus sympathiques parmi celles des grands mathématiciens du XVII^e siècle. (1) Né à Visé près de Liége, le 2 juillet 1622, il fit ses études à Louvain, puis à Rome, où il séjourna dix ans. Il rentra ensuite dans sa patrie, et occupa ses trop rares loisirs et même ses nuits à de profondes recherches mathématiques, qu'il poursuivit presque jusqu'à sa mort, survenue le 19 mars 1685.

C'était un homme d'une érudition formidable, doué, pour les mathématiques, d'un esprit d'invention et de généralisation très remarquable. Malheureusement, les lourdes fonctions ecclésiastiques qu'il remplissait dans un milieu « béotien » où il était absolument isolé, entravèrent l'essor de cette intelligence si féconde et si brillante. Cet isolement lui fut toujours pénible, et il n'est rien de plus mélancolique que les endroits de ses lettres où il se plaint de l'incompréhension de ses concitoyens et des difficultés qu'il éprouve à se procurer les livres qu'il rassemble avidement dans sa bibliothèque. Il ne pouvait remédier que partiellement à cette situation par la correspondance active qu'il

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(1) La seule source sérieuse pour l'étude de la vie et des œuvres de R. de Sluse est le remarquable travail de C. Le Paige : Correspondance de R. F. de Sluse, *Bull. Boncompagni*, t. 17, 1884.

Cf. aussi C. Le Paige : Un géomètre belge du XVII^e siècle : R. de Sluse, *Ciel et Terre*, t. 2, 1887 ; reproduit presqu'entièrement dans :

C. Le Paige, Notes pour servir à l'Histoire des Mathématiques dans l'ancien Pays de Liége, *Bull. Inst. Archéologique Liégeois*, t. 21, pp. 78-95, 1890.

C'est à ces mémoires que j'emprunte la plupart des détails cités, et que je renvoie pour leur justification. Je les citerai sous les dénominations respectives : [A], [B], [C].

entretenait avec les savants les plus en vue de son temps, surtout PASCAL, HUYGENS et OLDENBURG.

Malgré ces circonstances défavorables, les rares écrits qu'il publia lui valurent immédiatement une estime universelle, d'ailleurs pleinement méritée. Son étude de prédilection était la géométrie analytique ; la méthode d'analyse qu'il emploie lui est d'ailleurs personnelle, et ne dérive nullement de celle de DESCARTES, mais bien de l'*Isagoge* (2) de VIÈTE, dont il reprend les notations. Sa plus belle invention dans ce domaine est sa méthode de résolution géométrique des équations, bien plus souple et plus élégante que celle de DESCARTES ; il la publia dans la seconde édition de son *Mesolabum* (3), et elle eut un grand retentissement. Quant à sa méthode de détermination des tangentes aux courbes, la seule chose qu'il se décida à publier (4) outre son *Mesolabum*, je me propose précisément de l'examiner ici et d'essayer d'apprécier l'influence qu'elle put avoir sur l'élaboration des principes du Calcul différentiel.

Je commencerai par analyser une autre solution du même problème, due à SLUSE et retrouvée dans ses papiers manuscrits (5): elle a été exposée pour la première fois par C. LE PAIGE (6) ; néanmoins il m'a paru utile de publier *in extenso* le texte original.

Les papiers de SLUSE contiennent encore un texte qui intéresse notre sujet, et qui est également publié ici : c'est un exposé détaillé de la démonstration de sa méthode analytique, dont il n'avait donné dans les *Philosophical Transactions* qu'un schéma insuffisant pour permettre de porter un jugement exact sur sa valeur.

L'ordre dans lequel les deux méthodes sont considérées ici correspond à l'ordre dans lequel les papiers qui s'y rapportent

(2) F. VIETAE *Opera mathematica*, ed. VAN SCHOOTEN, Ludg. Batav., 1646 : In artem analyticen isagoge ; cf. p. 8, De legibus Zeteticis, 5.

(3) R. F. SLUSII *Mesolabum*, 1e édition, Liége, 1659; 2e éd. avec les suppléments *De Analysi* et *Miscellanea*, Liége, 1668.

(4) *Philos. Transact.*, vol. 7, 1672, pp. 5143-5147; vol. 8, 1673, p. 6059. Quelques extraits de lettres de SLUSE sur le problème d'ALHAZEN ont été également publiées dans les *Philos. Trans.*, vol. 8, 1673, pp. 6119-6126, 6140-6146. Toutes ces lettres sont reproduites dans le recueil de LE PAIGE, ainsi que dans les *Œuvres Complètes de* HUYGENS, éditées par la Société hollandaise des Sciences (v. Index).

(5) Ces papiers sont réunis en plusieurs volumes conservés à la Bibliothèque Nationale, fonds latin, nos 10159, 10197, 10247, 10248, 10249, 10250, 10254. Ces manuscrits sont décrits dans LE PAIGE, [A], pp. 54-60 du tirage à part.

(6) LE PAIGE, [B] (non reproduite dans [C].)

sont classés dans le manuscrit. Mais ces papiers ne sont pas datés, et le classement (d'ailleurs très imparfait) des manuscrits semble être plutôt un classement par matières. Cependant nous avons des données plus précises qui nous permettent, sinon de dater avec certitude les deux méthodes, du moins de rétablir leur succession chronologique probable. C'est ce que nous montrerons plus loin.

II. Méthode cinématique.

La première solution du problème des tangentes donnée par DE SLUSE est au fond celle de ROBERVAL (7) et de TORRICELLI (8) : on considère la courbe donnée comme le lieu d'un point animé à la fois de deux mouvements de vitesses connues à chaque instant; la direction de la tangente à un instant donné est alors celle de la vitesse résultante à cet instant.

Nous trouvons une première allusion à cette méthode dans la lettre à PASCAL du 2 août 1658 : après avoir traité un élégant problème de quadrature des cycloïdes généralisées (9), DE SLUSE ajoute : « Je voudrois bien sçavoir si vous avez aussi considéré les touchantes de toutes ces infinies cycloïdes, lesquelles se peuvent tirer d'une maniere universelle et tres facile. » Il s'agit bien de la méthode qui nous occupe : car l'exposé qui est publié ci-après (*Appendice I*) commence justement par la construction de la tangente aux cycloïdes, et est précédé immédiatement, dans le manuscrit, par les problèmes de quadrature relatifs à ces mêmes courbes.

Si, comme il est probable, les deux problèmes ont été traités vers la même époque, il faut faire remonter plus haut encore leur solution : dans la lettre à PASCAL du 6 juillet 1658, nous trouvons en effet ces mots : « Quant est du premier [problème] qui touche la mesure de la Cycloïde et de ses parties, je lavois desja resoud il y a longtemps dune maniere universelle... »

Il est certain que DE SLUSE n'a pas été influencé par ROBER-

(7) ROBERVAL, *Observations sur la composition des mouvements, etc.*, Divers Ouvrages de Mathématique et de Physique, 1693. pp. 69 sqq.

(8) TORRICELLI, *Opera geometrica*, Florentiae 1644, pp. 120 sq.

(9) Cette généralisation, qui paraît due à DE SLUSE [Lettre à PASCAL du 2 août 1658 ; cf. Lettre au même du 6 juillet 1658], est la suivante : la cycloïde est la trajectoire d'un mobile qui parcourt d'un mouvement uniforme une courbe quelconque entraînée dans une translation uniforme.

VAL (10), car il n'a eu connaissance des travaux de ce dernier que par l'*Histoire de la Roulette*, que lui envoya PASCAL. Il dit dans sa réponse, datée du 16 novembre 1658 : « Javois escrit quelque chose dans mes lettres (sil m'en souvient) de la methode universelle de tirer les touchantes aux Roulettes qui n'at autre principe que celui des mouvemens dont elles sont composees, ce qui sestend aussi a infinité d'autres lignes, et j'en croyois estre le premier auteur. Mais je vois que Mr. DE ROBERVAL en avoit desja produit une toute semblable ou peut estre la mesme..... je souhaiterois passionement que ces grands hommes donnassent au public leurs inventions pour oster le soupçon destre plagiaires a ceux qui pourroient tomber par hazard dans les mesmes pensers. »

Mais l'influence de TORRICELLI est au contraire très nette, comme l'a déjà fait observer LE PAIGE (11). Ceci s'explique aisément par le séjour prolongé du savant liégeois en Italie (1642-1651) : sans doute faut-il placer à cette époque même les travaux qui nous occupent. Il est très instructif de comparer les deux méthodes : la comparaison met en relief cet esprit de généralisation et de systématisation qui est un des traits caractéristiques du talent mathématique de SLUSE ; pour lui, généraliser les méthodes doit être la préoccupation dominante du géomètre : « cum infinita propemodum theorematum et Problematum seges sit, in methodos incumbendum esse, quibus plurima generis ejusdem solvuntur. » [Lettre à HUYGENS du 31 juillet 1657].

TORRICELLI (*loc. cit*) traite en détail le cas de la parabole (C'est en effet à l'occasion de l'étude du mouvement des projectiles pesants qu'il a été amené à la découverte de sa méthode), puis indique brièvement que le principe de composition des mouvements s'étend à toutes les courbes. L'exemple de la cycloïde s'offrait tout naturellement, et c'est le procédé de TORRICELLI que DE SLUSE applique immédiatement aux cycloïdes plus générales qu'il considère [cf. *Appendice I*, « De tangentibus cycloidum »]. Mais où DE SLUSE va plus loin que TORRICELLI, c'est lorsqu'il essaye d' « embrasser dans une seule description » tous les mouvements accélérés, ou du moins ceux qu'il appelle « uniformément accélérés ».

(10) La méthode de ROBERVAL a été connue dès 1644, à peu près en même temps que celle de TORRICELLI, par l'intermédiaire de MERSENNE, *Ballistica et Acontismologia*, Parisiis, 1644, pp. 115-116. (Je cite d'après LE PAIGE, [Ç]).

(11) LE PAIGE, [B], [C].

Comme nous venons de le rappeler, TORRICELLI s'occupait du mouvement « uniformément accéléré » au sens moderne du mot, ou, si on veut, au sens de GALILEE. Il introduit dans ses raisonnements des artifices ingénieux du genre suivant : un point pesant tombe d'un point A en un point B ; en B, il continue à se mouvoir horizontalement avec la vitesse acquise dans sa chute, et d'un mouvement uniforme. Il utilise notamment la propriété que possède un tel mobile de parcourir dans le temps de la chute AB un espace horizontal BC = 2.AB. Si l'espace x est parcouru dans le temps t avec l'accélération constante g, on a en effet

$$t \frac{dx}{dt} = 2 x$$

à condition que le mobile parte du repos ($x = o$ et $\frac{dx}{dt} = o$ pour $t = t_0 = o$).

Or c'est cette propriété caractéristique du mouvement uniformément accéléré que DE SLUSE généralise en remplaçant le coefficient 2 de la formule précédente par le facteur quelconque k. Sa définition est présentée d'une manière qui rappelle tout à fait celle de TORRICELLI [cf. *Appendice I*, « Applicatio eiusdem methodi... »].

Les mouvements « uniformément accélérés » au sens de SLUSE sont donc définis par l'équation

$$t \frac{dx}{dt} = k x,$$

ou en intégrant,

$$x = a t^k,$$

a étant une constante arbitraire. On remarque combien cette conception est défectueuse du point de vue dynamique ; DE SLUSE était loin d'être un physicien. (12)

(12) M. LE PAIGE me paraît exagérer la valeur des travaux de physique et de mécanique du savant chanoine, et surtout de ses recherches sur le choc des corps. Sans doute cet esprit curieux de tout, élevé à l'école de GALILÉE, a-t-il pris plaisir à faire des observations de physique et d'histoire naturelle, à construire des appareils ingénieux, à produire des réactions chimiques, mais il n'a conduit aucune investigation expérimentale proprement dite. Il n'est pas surprenant non plus qu'il ait adopté le système de COPERNIC, toujours grâce à l'influence italienne.

Il était anti-péripatéticien [cf. Lettre à HUYGENS du 13 octobre 1664 : « Rationem continuo reddet Peripateticus quispiam... Sed plane contrarium contigit »] et grand admirateur de DESCARTES [cf. entre autres la Lettre à HUYGENS du 14 août 1657 : « viri incomparabilis autoritas »]. Mais les citations suivantes montreront qu'il avait fort mal profité des leçons de TORRICELLI : « Accedit etiam experientia,

Mais au point de vue analytique, la chose est très intéressante. En combinant les deux dernières formules, on a immédiatement (en posant $a = 1$)

$$\frac{d}{dt}(t^k) = k\,t^{k-1},$$

et cette formule est valable *quel que soit* k, c'est-à-dire même si k n'est pas entier positif. Disons dès maintenant que la seconde méthode de SLUSE aboutissait précisément à cette formule, mais seulement pour les valeurs entières et positives de k.

De plus, dans l'exemple d'application de la méthode, donné par DE SLUSE dans le manuscrit, un des deux mouvements composants est uniforme, de sorte qu'on peut considérer la coordonnée correspondante comme représentant le temps (13), ce qui permet de passer aisément d'une méthode à l'autre.

C'est ce qu'observe M. LE PAIGE [B] : « la vitesse ou fluxion de l'ordonnée reste constante, tandis que celle de l'abscisse est variable. On voit combien il est regrettable que SLUSE n'ait pas poursuivi son idée. S'il l'avait combinée avec sa [seconde méthode], il aurait vu que sa conclusion subsistait pour une valeur quelconque de [k], car dans sa méthode mécanique des tangentes, le rapport

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ex qua sola ut pronuntiandum non existimo, ita contemnendam non censeo cum rationi adstipulatur. » [Lettre à HUYGENS du 19 octobre 1657] « Nec est quod experientias sequaris, quibus ut fidem non abrogo, ita mihi semper Coi senis occurrit illud *ἡ πεῖρα σφαλερή, ἡ δὲ κρίσις χαλεπή*, nisi ratio confirmet. Scis enim quid *τὰ ἔξωθεν* hac in re possint. » [Lettre à HUYGENS du 18 décembre 1657].

Pour ce qui concerne les lois du choc des corps, M. LE PAIGE suggère ([A], [B], [C]) que DE SLUSE a pu les découvrir indépendamment de HUYGENS et avant lui. Mais la publication intégrale de la correspondance de HUYGENS nous oblige à rejeter absolument cette suggestion : il ressort de ces documents que le savant hollandais a découvert ses lois d'une manière indépendante, après avoir reconnu la fausseté de celles de DESCARTES ; le 20 juillet 1656, il écrit à ROBERVAL : « J'ay achevé mon petit ouvrage depuis peu de jours. » [*Œuvres*, t. II, No 315]. Quant aux lois énoncées par DE SLUSE [Biblioth. Nation., fonds latin, ms. 10248, fol. 61-62], elles sont absolument fausses : il y est fait allusion dans quelques lettres échangées avec HUYGENS en 1657 : HUYGENS remarque que les lois de SLUSE doivent être différentes des siennes, et conduisent à des résultats différents, contrairement à ce qu'affirme M. LE PAIGE ([C], p. 94). Il semble que M. LE PAIGE, n'ayant pas sous les yeux tous les documents nécessaires, a interprété dans un sens trop large un passage amphibologique de la Lettre à HUYGENS No 30 (fin 1657).

(13) Cf. NEWTON, *Méthode des fluxions*, trad. BUFFON, Paris 1740, p. 21 : « je supposerai que l'une des Quantités proposées... doit augmenter par une Fluxion uniforme, à laquelle Quantité je rapporterai tout le reste comme si c'étoit au tems... »

est quelconque, et il aurait pu appliquer son procédé analytique aux courbes dont l'équation contient des fractions ou des radicaux. »

Il faut toutefois remarquer que cette conclusion suppose DE SLUSE en possession de la théorie des exposants non entiers positifs, théorie dont je n'ai trouvé aucune mention ni application dans tout l'œuvre de notre savant. Cependant ,il n'est pas interdit de penser que s'il avait suivi la voie imaginée par M. LE PAIGE, il eût été capable d'en voir la nécessité et de la retrouver comme NEWTON et LEIBNIZ.

Mais la raison principale qui a détourné DE SLUSE de ce rapprochement fécond est sans doute qu'il s'est cru devancé par ROBERVAL, ce qui l'a engagé à cesser ses recherches de ce côté. Plus tard, au moment où il a trouvé sa seconde méthode, il avait perdu de vue la première, et n'a pas pensé à établir entre les deux une comparaison approfondie.

III. Méthode analytique.

Pour essayer de reconstituer la genèse de la méthode analytique de SLUSE, nous avons à notre disposition les deux passages suivants de sa correspondance :

1° Lettre à HUYGENS du 12 janvier 1663 :

« Gaudio Te ac Clarissimum HUDDENIUM in tangentium methodum meae non absimilem incidisse... (14) Ego meam do ere possum vel omnino rerum Geometricarum rudem ; brevissimis enim regulis adjutus, quamlibet propositam aequationem, levi aliquot characterum mutatione in eam quae tangentem indicat, absque ullo calculo convertet. Hanc vero ex ea deduxi qua decennio et amplius usus fueram, sed non absque aliqua calculi molestia ; in quem dum attentius inspicerem, methodum tandem inveni qua faciliorem arbitror dari non posse. »

2° Lettre à OLDENBURG du 22 novembre 1670 :

« ... in Clar.^{mi} BARROVII Lectionibus, (15)...., praeclara multa et auctore digna observavi, et non mediocriter in primis sum gavisus, eandem ipsi occurrisse ducendarum tangentium methodum, qua olim usus fueram. Verum in aliam mecum incidet

(14) Cf. HUYGENS, *Regula ad inveniendas tangentes linearum curvarum*, Divers Ouvrages de Math. et de Phys., Paris, 1693, pp. 330 sqq.

(15) J. BARROW, *Lectiones Opticae et Geometricae*, Lond. 1674, p. 80 de la 2de pagination (1re édition 1670).

longe faciliorem, et quae vix ullam calculi molestiam requirat, si paulo ulterius eadem via progressus fuerit. »

On voit donc que la découverte de SLUSE s'est faite en deux étapes : dès la fin de son séjour en Italie, il était en possession d'une méthode analogue à celle de BARROW ; c'est vers cette même époque, comme nous l'avons vu, qu'il a trouvé aussi sa première méthode ; mais ce n'est que plus tard qu'il a déduit de la méthode de BARROW la règle d'une grande simplicité qui constitue à proprement parler sa seconde solution du problème. Il est malaisé de déterminer à quel moment s'est effectué le passage de l'un à l'autre stade. Dans la lettre à PASCAL du 29 juin 1658, il donne une construction de la tangente aux « perles », qui se déduit manifestement de sa règle analytique ; et dans la lettre précédente (6 avril 1658), où il avait annoncé cette construction, il disait : « Il y a longtemps que j'avois considere vos lignes en perle... ». Malheureusement, il est impossible de savoir si c'est de la première ou de la seconde forme de la règle analytique qu'a été déduite la construction en question. En somme, comme nous allons le voir, la seconde forme n'est qu'une systématisation de la première, mais cela n'implique pas qu'elle ait été obtenue immédiatement après. Au contraire, le premier texte reproduit plus haut (« methodum *tandem* inveni ») semble indiquer qu'un certain temps a séparé les deux découvertes. De tout ceci, il résulte qu'on peut placer vers 1655 ou 1660 l'établissement de la règle universelle que nous allons analyser.

La méthode de BARROW (15), rappelons-le, est en principe la suivante. Soit

$$f\ (x,\ y) = o$$

l'équation d'une courbe. Si $(x + e,\ y + a)$ est un point de la courbe infiniment voisin du point $(x,\ y)$, on peut écrire

$$f\ (x + e,\ y + a) = o. \qquad (1)$$

Dans le calcul du premier membre, on peut négliger les termes indépendants de e et de a, en vertu de $f\ (x,\ y) = o$, et les termes d'ordre supérieur au premier en e et a : « etenim, dit BARROW, isti termini nihil valebunt ». On trouve ainsi

$$e\ f'_x + a\ f'_y = o, \qquad (2)$$

d'où on déduit la valeur de la sous-tangente S en remplaçant a par y et e par S.

L'inconvénient de cette méthode est évidemment la « calculi

molestia » qui résulte de ce que le calcul à exécuter pour arriver à l'équation (2) dépend de la forme particulière de la fonction $f(x, y)$. Pour y remédier, il était naturel de chercher une loi de formation des termes f'_x, f'_y de l'équation (2), permettant d'écrire cette équation directement, sans passer par le développement explicite de l'équation (1). C'est essentiellement en cela que consiste le progrès réalisé par DE SLUSE : on reconnaît bien là encore une fois la marque de l'esprit systématisateur du savant liégeois.

Essayons de nous rendre compte de la manière dont il a pu être conduit à cet important perfectionnement. Pour cela, reprenons d'abord, en notations modernes, la démonstration dont le texte original est reproduit dans l'Appendice II.

Posons $f(x, y) = \Sigma\, a_{pq}\, x^p\, y^q$,

et $x + e = x_1,\ y + a = y_1$.

Le premier Lemme de SLUSE s'exprime par la formule

$$\frac{x_1^p - x^p}{x_1 - x} = x_1^{p-1} + x_1^{p-2}x + \ldots + x_1 x^{p-2} + x^{p-1}$$

Le second Lemme consiste à dire que le second membre de cette formule contient p termes. Nous examinerons séparément plus loin le troisième Lemme.

Du Lemme I, on déduit,

$$\begin{aligned} o = f(x_1, y_1) - f(x, y) &= \Sigma\, a_{pq}\,(x_1^p\, y_1^q - x^p\, y^q) \\ &= \Sigma\, a_{pq}\,[x_1^p\,(y_1^q - y^q) + y^q\,(x_1^p - x^p)] \\ &= (y_1 - y)\,\Sigma\, a_{pq}\, x_1^p\,(y_1^{q-1} + \ldots + y^{q-1}) \\ &\quad + (x_1 - x)\,\Sigma\, a_{pq}\, y^q\,(x_1^{p-1} + \ldots + x^{p-1}) \quad (3) \end{aligned}$$

Si A est le pied de l'ordonnée du point D (x, y), et si C est le point de rencontre de l'axe des x avec la sécante joignant les points D et F (x_1, y_1) de la courbe, posons (abstraction faite du signe)

$$AC = S\ ;$$

quel que soit F (au voisinage de D), on a

$$\frac{y_1 - y}{x_1 - x} = \frac{y}{S}\ ;$$

lorsque F coïncide avec D, le premier membre devient indéterminé, mais le second a une valeur finie y : S, S désignant maintenant la sous-tangente. En tenant compte de cette remarque, nous pouvons écrire (3) sous la forme

$$y\Sigma\, a_{pq}\, x_1^p\,(y_1^{q-1} + \ldots + y^{q-1}) + S\,\Sigma\, a_{pq}\, y^q\,(x_1^{p-1} + \ldots + x^{p-1}) = o\ ;$$

pour avoir la valeur S de la sous-tangente, il suffit maintenant

de faire dans cette équation

$$x_1 = x \text{ et } y_1 = y,$$

ce qui est permis d'après ce qui vient d'être dit. Cela donne

$$y \Sigma a_{pq} x^p . qy^{q-1} + S \Sigma a_{pq} y^q . p x^{p-1} = o,$$

d'après le Lemme II. La règle se déduit immédiatement de cette dernière formule.

Examinons maintenant point par point cette élégante démonstration, en essayant de la raccorder aux considérations de BARROW. Puisque, dans le développement de la formule (1), les termes indépendants de e et a sont évidemment $f(x, y)$, le premier membre de la formule (2) peut s'écrire, en faisant abstraction des termes d'ordre supérieur

$$e f'_x + a f'_y = f(x + e, y + a) - f(x, y) \qquad (4)$$

A ce moment intervient la première remarque fondamentale de SLUSE, résumée dans son Lemme I (auquel il adjoint le Lemme accessoire II) : c'est qu'on peut transformer le second membre en un aggrégat de termes contenant chacun un facteur de la forme $y_1^q - y^q$ ou $x_1^p - x^p$, et que les expressions de cette forme sont justement divisibles respectivement par $y_1 - y$ et par $x_1 - x$. Ainsi, il est possible d'écrire *rigoureusement* une équation de la forme (4) :

$$f(x_1, y_1) - f(x, y) = e \phi_1 (x, y ; x_1, y_1) + a \phi_2 (x, y ; x_1, y_1)$$

Les fonctions ϕ_1, ϕ_2 doivent se réduire respectivement à f'_x et f'_y lorsque e et a tendent vers zéro. Or la forme de ϕ_1 et ϕ_2 est connue, donc la forme de f'_x et f'_y s'en déduira aisément en y faisant $x_1 = x$, $y_1 = y$, et le problème est résolu.

Le remplacement de ϕ_1 et ϕ_2 par f'_x et f'_y revient justement à négliger les infiniments petits d'ordre supérieur. C'est ce que faisaient couramment les mathématiciens contemporains, sans se donner la peine de justifier (16) ce procédé. Il est très remarquable de voir que DE SLUSE se montre plus soigneux, et ce point donne à sa démonstration un intérêt vraiment unique, encore que la manière dont il tourne la difficulté soit fort artificielle (17). C'est

(16) On sait combien il est fâcheux que NEWTON n'ait pas traduit en langage analytique, à l'usage de ses disciples moins perspicaces, les idées parfaitement claires et précises sur les limites qu'il expose sous forme géométrique au début des *Principia*.

(17) La même idée, sous une forme assez différente, se trouve à la base de la curieuse tentative d' « arithmétisation » de la théorie des fluxions esquissée par JOHN LANDEN, *A Discourse concerning the Residual Analysis*, London, 1758.

justement ce second point important que notre auteur « dissimule » véritablement dans l'énoncé d'un Lemme III tout à fait trivial : « Si deux quantités égales sont divisées par deux autres de rapport donné, les quotients sont en raison inverse de ce rapport ». (18) Il utilise ce lemme pour remplacer $x_1 - x$ et $y_1 - y$ respectivement par S et y de manière à lever l'indétermination du rapport $x_1 - x : y_1 - y$, et ce n'est qu'*après* cette substitution (contrairement à ce que faisait BARROW) qu'il passe à la limite : ϕ_1 et ϕ_2 tendent respectivement vers f'_x, f'_y et S tend vers la valeur de la sous-tangente. La formule

$$S f'_x + y f'_y = o$$

est ainsi établie d'une manière rigoureuse.

Après avoir étudié la découverte de SLUSE en elle-même, il nous reste à discuter ses rapports avec les travaux de NEWTON et de LEIBNIZ. Pour ce qui concerne le premier, la chose est mise au point depuis longtemps (19) : les deux géomètres sont arrivés à leurs résultats respectifs indépendamment l'un de l'autre ; si DE SLUSE a trouvé sa méthode avant NEWTON, ce dernier l'a énoncée sous une forme plus générale (applicable aux fractions et radicaux), et cela simplement comme application de sa méthode des fluxions, dont le domaine était infiniment plus vaste ; il ne peut être question de poser le modeste mathématicien liégeois en rival de NEWTON ou de LEIBNIZ (20), mais seulement d'estimer le rôle qu'a pu jouer la publication de sa méthode dans l'élaboration des géniales découvertes des deux illustres savants.

Cette question est plus délicate à résoudre dans le cas de LEIBNIZ. On sait que NEWTON, dans la *Recensio libri qui inscriptus est Commercium epistolicum* (21), insinue, en commentant une lettre de

(18) L'artifice est si bien dissimulé de cette façon qu'il a échappé à ZEUTHEN (*Geschichte der Mathematik im XVI. u. XVII. Jahrhundert*, Leipzig, 1903, S. 337), parce qu'il n'avait pas sous les yeux le texte manuscrit qui aurait pu le guider. ZEUTHEN donne, à la fin de son essai de reconstitution de la démonstration, une interprétation inexacte du Lemme III.

(19) LE PAIGE, [A], p. 53 ; *Commercium epistolicum*, éd. de 1722, pp. 104-107.

(20) J'estime exagérée l'appréciation suivante de M. GILBERT, citée non sans complaisance par M. LE PAIGE, [B], p. 14 du tirage à part : « Cette méthode suffit pour autoriser la conjecture que, placé dans un milieu scientifique plus intense, SLUSE eût été capable de ravir à LEIBNIZ et NEWTON l'honneur de leur sublime découverte. »

(21) Cf. p. ex. l'édition du *Commercium epistolicum*, citée dans la note (19) p. 50 et p. 109.

LEIBNIZ à OLDENBURG (du 18/28 novembre 1676), que LEIBNIZ n'aurait commencé qu'après cette date à s'occuper de perfectionner la méthode de SLUSE de manière à la rendre applicable aux fractions et aux radicaux. La publication par GERHARDT (22) de certains manuscrits de LEIBNIZ permet de remettre les choses au point et de préciser le rôle qu'a pu jouer la méthode de SLUSE dans les travaux du rival de NEWTON.

Le 11 novembre 1675, LEIBNIZ intègre une équation différentielle et vérifie le résultat en lui appliquant la méthode de SLUSE pour retrouver l'équation initiale : il n'avait donc pas encore, à cette date, effectué l'application de son « novum calculi genus » au problème des tangentes ; émerveillé des résultats qu'il venait d'obtenir dans des problèmes de quadratures, il était pressé de s'attaquer de suite, pour éprouver la puissance de sa méthode, aux difficiles problèmes qu'on désignait sous le nom de « methodus tangentium inversa ». Mais dès le 26 juin 1676, il avait commencé l'étude de la « methodus tangentium directa » et constaté qu'elle était contenue comme les autres dans son calcul différentiel : « vera methodus tangentium generalis est per differentias. Ut scilicet ordinatarum (directarum vel convergentium) quaeratur differentia. »

Cependant, il ne s'arrête pas encore au développement de cette idée, mais revient à des problèmes d'un autre genre, comme le problème de DE BEAUNE, qui lui permettent de perfectionner son algorithme. On voit donc qu'à son départ de Paris, en octobre 1676, il était pleinement conscient de toute la portée du nouveau calcul, bien qu'il fût loin d'en avoir arrêté tous les détails ; en ce qui concerne le point qui nous occupe, il savait que le problème des tangentes était susceptible d'une solution générale dont il avait le principe, mais il n'avait pas examiné le détail du calcul.

A ce moment, lors de son passage à Amsterdam, il eut connaissance de la méthode des tangentes de HUDDE, que celui-ci n'avait pas publiée, et fut surpris d'apprendre que cette méthode, plus perfectionnée que celle de SLUSE, permettait souvent de calculer la sous-tangente sans devoir préalablement rendre l'équation rationnelle. C'est ce qu'il dit justement dans cette fameuse lettre du 18/28 novembre, qui a été si tendancieusement interprétée

(22) Pour tout ce qui suit, cf. GERHARDT, *Die Entdeckung der höheren Analysis* Halle, 1855.

par NEWTON. La surprise de LEIBNIZ à cette occasion, ainsi que la manière dont il insiste plus tard sur la limitation de la méthode de SLUSE aux équations rationnelles, permettent de conjecturer que c'est à ce moment seulement que son attention fut attirée sur cette imperfection, et qu'il fut dès lors tout naturellement conduit à étudier de près comment se comportait à cet égard son propre procédé. Il arrive immédiatement à un résultat tout à fait satisfaisant, car dès la fin du mois il en note un exposé rapide, qu'il termine en remarquant que la méthode s'applique aux radicaux, que les coordonnées ne doivent pas être nécessairement cartésiennes, et enfin que la méthode n'est pas limitée au seul cas de deux variables.

En résumé, jusqu'à son entrevue avec HUDDE, LEIBNIZ n'a connu que la méthode de SLUSE ; il l'a notamment appliquée à la vérification des premiers résultats qu'il a obtenus par son nouveau calcul. Lorsqu'il s'est aperçu de la limitation de cette méthode aux équations rationnelles, il a immédiatement cherché à la perfectionner par l'emploi de son algorithme. Telle est, je crois, la place qu'il convient d'assigner à DE SLUSE dans l'histoire de la découverte leibnizienne : une place bien modeste, bien effacée si on la compare à celle de son génial contemporain, mais qui prend cependant une importance considérable quand on envisage plutôt l'histoire de la période précédente, celle des précurseurs.

Parmi ceux-ci, c'est un rang très honorable qu'il faut réserver à DE SLUSE, car il eut le mérite de donner le premier un algorithme déjà très général permettant de déduire sans calcul de la forme analytique d'une fonction la forme de la dérivée. Cette étape a été essentielle dans la découverte de NEWTON comme dans celle de LEIBNIZ ; et le fait d'en avoir reconnu l'importance est tout à l'honneur du savant liégeois.

(*Liége*) L. ROSENFELD.

APPENDICE

I. *Bibliothèque Nationale*, fonds latin, nº 10247 fol. 89-90.

« De tangentibus cycloidum. »

« Ut omnium cycloidum unica demonstratione ostenditur quadratura (*) ita

(*) *Ibidem*, fol. 88.

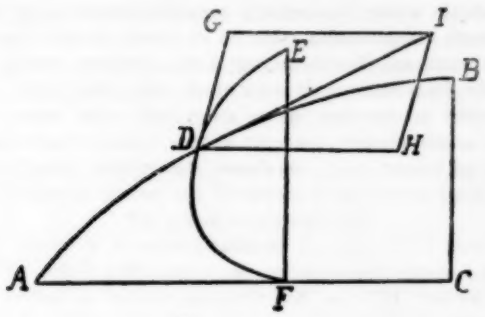

Fig. 1

etiam unica demonstratione ostenditur methodus ducendi tangentem in quolibet puncto. In semicycloide ADB datum sit punctum D & intelligatur descripta figura genitrix per illud transiens EDF ad quam ducetur tangens GD, et ex puncto D resecetur DG quolibet intervallo, fiatque ut velocitas qua percurritur curva FDE ad velocitatem qua percurritur recta AC ita GD ad DH parallelam ipsi AC et compleatur parallelogrammum DGIH. Dico ductam dimetientem ID tangere cycloidem in D puncto. Demonstratio facilis est, cum enim DG tangat curvam EDF, est directio puncti D iuxta motum per curvam FD. recta vero DH est directio motus eiusdem puncti D παραλληλῶς lineae AC estque GD ad DH ratio velocitatum. Igitur si punctum D promovetur (?) ysdem velocitatibus versus G & versus H, describet rectam DI. Sed punctum D in cycloide movetur motu composito ex ijsdem velocitatibus et directionibus. Igitur directio puncti G [*sic*] in cycloide est eadem recta DI. hoc est recta DI tangit cycloidem. Quod erat D.

« Eadem demonstrari possunt methodo Archimedis et antiquorum Geometrarum per deductionem ad impossibile cum in quolibet cycloidis puncto varietur directio sicuti variatur inclinatio tangentis curvam FDE. Quod Lectori Geometrae patebit.

« Applicatio eiusdem methodi ad tangentes earum linearum quarum motus componentes novimus. »

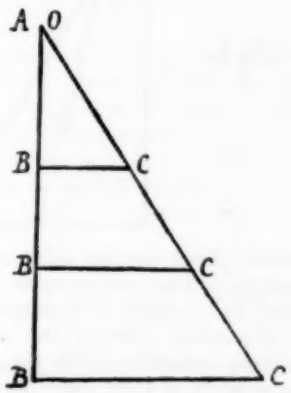

Fig 2

« Eadem methodo etiam inveniemus tangentes omnium curvarum quae ex motibus nobis notis componuntur (sive illi in natura dentur sive non) sive sint aequabiles, sive accelerati. Acceleratorum autem infinitae species esse possunt. Accelerantur enim iuxta rationes diversas quod unica descriptione complectemur. Motum uniformiter acceleratum (de eo enim solo nobis sermo) dicimus eum, qui impetu per accelerationem concepto eodem tempore motu aequabili lineam percurrit, quae ad lineam motu accelerato descriptam habeat semper eandem rationem. & exempli gratia cadat mobile O per rectam normalem AB motu ac-

BC debet esse maior AB

celerato, ita scilicet (?) ut sumpto quolibet puncto B si mobile O cum impetu in B acquisito moveretur horizontaliter per rectam BC, eodem tempore quo descripsit AB percurreret BC ipsa AB maiorem semper in eadem ratione dupla, tripla, ... [3 mots illisibles.]

« His ita praemissis facile est invenire tangentem ex methodo superiori. feratur exempli gratia punctum A per rectam AB motu aequabili, et eodem tempore

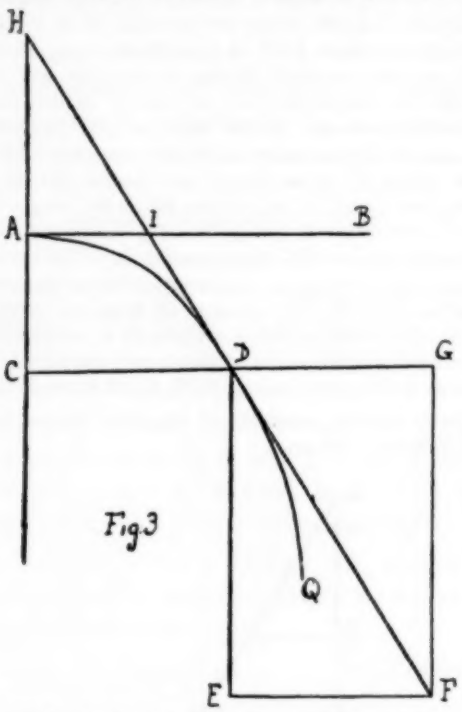

Fig 3

per rectam AC quovis angulo priori iunctam motu accelerato iuxta quenlibet rationem et quaeratur tangens in quolibet puncto curvae descriptae, ut D. Ducatur DE parallela AC et demissa DC parallela AI productaque in G sumatur GD aequalis CD, ob motum aequabilem, itemque DE quae sit aequalis ei lineae quam describeret mobile motu aequabili eodem tempore quo motu accelerato descripsit AC cum impetu acquisito in C, et perficiatur parallelogrammum DFE Dico eiusdem dimetientem DF tangere curvam in D. Demonstratio eadem est quae prius, est enim DE directio motus puncti D per impetum a motu accelerato impressum, DG vero eiusdem directio per motum aequabilem impressum, sed

ex illis motibus componitur motus puncti D estque ratio impetuum eadem quae DE ad DG. Igitur eiusdem puncti D directio erit DF hoc est tangens. Quod erat D.

« Si motus per AB non esset aequabilis sed acceleratus tunc DG non esset sumenda aequalis CD sed maior iuxta rationem accelerationis ut fecimus de DE respectu AC. . . . . [Exemple de la parabole] . . . . . .

II. *Bibliothèque Nationale*, fonds latin, nº 10247, fol. 247-253.

« Demonstratio Regulae generalis ducendi tangentes ad quaslibet curvas Geometricas. »

« Sit quaelibet curva BD, cuius puncta per aequationem referantur ad rectam quamlibet EAB (sive sit axis sive diameter sive quaelibet alia, parum refert). BA in terminis analyticis perpetuo dicatur y. Applicata ex puncto D ad quod ducenda est tangens sit x et ducta tangente DC occurrente EB productae in C sit AC a. Regula haec est.

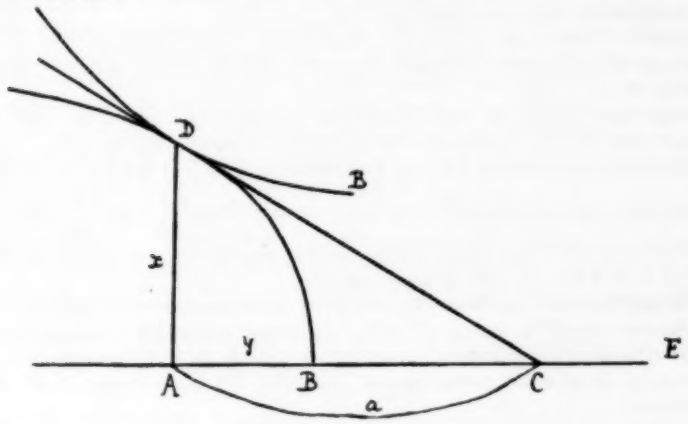

Fig 4

« Abyciantur ab aequatione omnes quantitates in quibus y vel x non reperitur. et statuantur ab uno latere partes in quibus y et ab altero partes in quibus x cum suis signis + vel —. Tum praefigatur unicuique parti numerus qui est exponens dignitatis quam habet y vel x. Demum in latere in quo sunt partes in quibus y, unum y in singulis partibus vertatur in a. Illa erit aequatio tangentis. Non enim ignorari poterit a cum ex hypothesi dentur y et x.

« Exempli gratia sit BD arcus circuli cuius diameter EB ‖ b. erit aequatio $by - yy \parallel xx$. itaque ex regula $ba - 2ya \parallel 2xx$ et $\frac{2xx}{b-2y} \parallel a$ sive $\frac{xx}{\frac{1}{2}b-y} \parallel a$. quod vel elementis constat. Intelligatur nunc AB producta in E, sitque BD arcus Hyperbolae cuius latus transversum EB ‖ b, rectum ‖ z. erit ἀναλογισμος $b|z|by + yy|xx$ et aequatio $\frac{zby + zyy}{b} \parallel xx$. Itaque aequatio pro tangente erit $\frac{zba + 2zya}{b} \parallel 2xx$ et $\frac{xxb}{\frac{1}{2}zb + zy} \parallel a$.

« Sit quaelibet aequatio $q\,q + b\,y - y\,y \parallel x\,x$; quae pariter est ad circulum· abiecto qq, fit aequatio pro tangente ut prius $\frac{x\,x}{\frac{1}{2}\,b - y} \parallel a$.

« Unicum est observandum quando in aequatione partes reperiuntur compositae ex y et x, ut $y\,x$, $y\,y\,x$, $x\,x\,y$ et similes. illae enim bis sumendae cum suo signo, semel quidem cum illis in quibus y, et semel cum illis in quibus x.

« Exempli gratia sit aequatio $b\,x\,x - y\,x\,x \parallel y^3$. faciendum est $b\,x\,x - y\,x\,x \parallel y^3 + y\,x\,x$. Erit ut prius aequatio pro tangente $2\,b\,x\,x - 2\,y\,x\,x \parallel 3\,y\,y\,a + x\,x\,a$. Nam ex regula in partibus quae a latere x statuuntur y verti non debet in a sicut in illis reperiatur. et in partibus quae a latere y statuuntur, sicut in illis sit x, non debet praefigi illis alius exponens nisi qui convenit y. et unum y vertendum est in a.

« Ut hanc regulam demonstrem suppono 1° Lemma quod alibi ostendi, scilicet Differentiam duarum quarumlibet potestatum eiusdem gradus, applicatam ad differentiam Laterum, dare partes singulares gradus inferioris ex binomio laterum.

« Ut $a^3 - e^3$ applicatum ad $a - e$ dare $a\,a + a\,e + e\,e$, quae sunt partes singulares ,quadrati $a + e$. sic $a^4 - e^4$ applicatum ad $a - e$ dare $a^3 + a\,a\,e + a\,e\,e + e^3$, quae sunt partes singulares cubi ex $a + e$.

« Suppono deinde, tot esse partes singulares in Gradu inferiori quot unitates habet exponens dignitatis immediate superioris, tres nimirum in quadrato, quatuor in cubo &c.

« Suppono demum, si duae quantitates aequales applicentur ad duas alias quarum ratio data sit, quotientes fore reciproce in eadem ratione data.

« Ut si $q\,q\,n$ sit aequale $p\,b\,b$ et applicetur $q\,q\,n$ ad z et $p\,b\,b$ ad d. Sit autem ut z ad d ita a ad e. dico fore $\frac{qq\,n}{z}$ ad $\frac{p\,b\,b}{d}$ ut e ad a. Nam $\frac{q\,q\,n}{z}$ ad $\frac{p\,b\,b}{d}$ est, ductis omnibus in z et d, ut $d\,q\,q\,n$ ad $z\,p\,b\,b$,et ablatis aequalibus $q\,q\,n$ et $p\,b\,b$ ex hypothesi ut d ad z hoc est e ad a. Quod erat D.

« His suppositis sit quaelibet curva BFD ad quam ducenda sit tangens in puncto D. Ducatur quaelibet secans ut DFC. occurrens rectae EB (ad quam puncta curvae referuntur) in puncto C. et cadant ex punctis D et F normales DA, FG. Evidens est quod si DC fieret tangens, duae FG, DA coinciderent et AG abiret in nihilum.

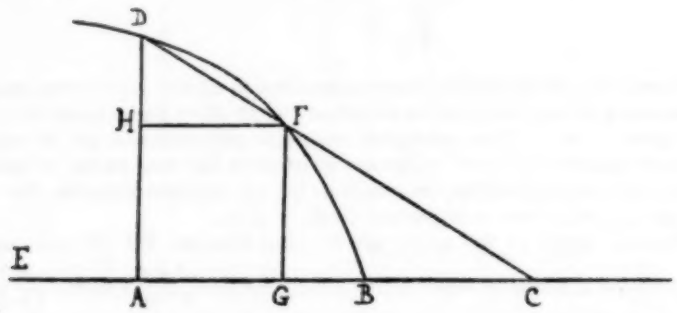

Fig. 5

« Evidens quoque est, ducta FH parallela AB, ita esse DH ad HF, ut DA ad AC.

« Nunc in terminis analyticis BA sit y, AD x, AC a, GB e, GF v, erit HF $y - e$,

et DH $x - v$: eritque ἀναλογισμος $x - v \,|\, y - e \,|\, x \,|\, a$, quaecumque tandem sit curva.

« Deinde, aequatio quae, ex proprietate curvae, ostendet eius relationem in puncto D ad rectam EB, erit omnino similis aequationi quae ostendit relationem eiusdem in puncto F ad eandem rectam. Nec alia erit differentia nisi quod in illa erit x, in hac eius loco v, in illa erit y et in hac eius loco e.

« Sit EG aequatio $b\,y - y\,y \parallel x\,x$ ad punctum D. erit ad punctum F $b\,e - ee \parallel vv$ Sic si fuerit $b\,x\,x - q\,x\,x \parallel y^3$ pro D erit $b\,v\,v - q\,v\,v \parallel e^3$ pro F. atque ita de singulis.

« Unde sequitur, si aequatio pro F, subtrahatur ab alia pro D, substrahendo scilicet quamlibet partem a qualibet sibi simili, omnes partes in quibus y et e tantum, posse semper dividi per $y - e$, et in quibus x et v, per $x - v$, quandoquidem perpetuo erunt dignitates eiusdem gradus applicatae ad differentiam radicum.

« Exempli gratia subtrahatur $b\,e - e\,e \parallel v\,v$, a, $b\,y - y\,y \parallel x\,x$, fiet $b\,y - b\,e - y\,y + e\,e \parallel x\,x - v\,v$ .patet autem $b\,y - b\,e$ et $- y\,y + e\,e$ dividi per $y - e$ et $x\,x - v\,v$ per $x - v$ cum sint dignitates eiusdem gradus applicatae ad differentiam radicum, ut diximus. vel etiam ipsa radicum differentia ducta in aliquam quantitatem cognitam, ut $b\,y - b\,e$, quae est $y - e$ in b. Atque hoc ultimo casu quotiens erit ipsa quantitas cognita, alys autem, partes singulares gradus inferioris, tot numero quot sunt unitates in exponente dignitatis y et e, aut x et v ut ostendimus. Quotientes autem erunt semper in ratione reciproca suorum divisorum. sic quotiens ex divisore $y - e$ ad quotientes ex divisore ex $x - v$, erit ut $x - v$ ad $y - e$, hoc est ut x ad a.

« Quoniam autem in puncto contactus y est aequalis e, et v aequalis x, sequitur pro e et v substituto eorum valore in quotientibus, totidem partes remansuras in ysdem quotientibus quot erant in una ex aequationibus ,sed singulas habituras numerum sibi praefixum qui Dignitati partis sibi respondentis in aequatione competebat.

« Exempli gratia $b\,y - b\,e - y\,y + e\,e \parallel x\,x - v\,v$ applicetur ab una parte ad $y - e$ ab alia ad $x - v$ erunt quotientes

$$b - y - e \;|\; \text{et} \;|\; x + v$$

et si y supponatur aequalis e et $x \parallel v$ fiet $b - 2\,y$ et $2\,x$. sic $b\,x\,x - q\,x\,x$ sit aequale y^3 et $b\,v\,v - q\,v\,v \parallel e^3$. fiet

$$y^3 - e^3 \parallel b\,x\,x - b\,v\,v - q\,x\,x + q\,v\,v$$

et facta applicatione

$$y\,y + e\,y + e\,e \quad \text{et} \quad b\,x + b\,v - q\,x - q\,v$$

et si $y \parallel e$ et $x \parallel v$, $3\,y\,y$ et $2\,b\,x - 2\,q\,x$ praefixo nimirum singulis partibus exponente dignitatis, quam obtinebat in aequatione dignitas superior ex qua oriuntur.

« Ostendimus autem quotientes semper futuros in ratione reciproca divisorum, sive quotientes ex divisore $y - e$, ad quotientem ex divisore $x - v$, ut x ad a.

« Unde ductis primis in a secundis in x fiet nova aequatio, non differens in alio a prima, nisi quod quaelibet pars in qua v, habeat praefixum exponentem dignitatis ipsius y, et unum y versum a. Quaelibet autem pars in qua x, sit omnino eadem nisi quod habeat exponentem gradus x sibi praefixum. Ut ex
$b - 2y \,|\, 2\,x \,|\, x \,|\, a$ fit $b\,a - 2\,y\,a \parallel 2\,x\,x$.
ex $3\,y\,y \,|\, 2\,b\,x - 2\,q\,x \,|\, x \,|\, a$ fit $3\,y\,y\,a \parallel 2\,b\,x\,x - q\,x\,x$.

« Et heac quidem, cum in aequatione proposita nulla pars est quae producta sit ex multiplicatione y et x vel graduum eorundem. Restat nunc ostendendum cur illas partes bis sumendas diximus in Regula.

« Si in duas quaslibet dignitates eiusdem gradus, ducantur duae aliae quaelibet dignitates homogeneae et ex productis fiat apotome, illa dividi non poterit per differentiam radicum vel priorum vel posteriorum dignitatum, exempli gratia si y^3 et e^3 ducantur illud in $x\,x$ et hoc in $v\,v$ fiet apotome $y^3\,x\,x - e^3\,v\,v$ quae dividi non poterit neque per $y - e$, neque per $x - v$. At si ab hac apotome auferas et eaedem addas productum ex prima dignitate in quartam, vel ex 2da in 3am, invariata primae apotomes quantitate, duas efficies, quarum una per $y - e$ alia per $x - v$ divisibilis erit. ut si $y^3\,x\,x - e^3\,v\,v$ addatur et auferatur productum ex y^3 in $v\,v$ vel ex e^3 in $x\,x$ fiet $y^3\,x\,x - y^3\,v\,v + y^3\,v\,v - e^3\,v\,v$ vel $y^3\,x\,x - e^3\,x\,x + e^3\,x\,x - e^3\,v\,v$. Patet autem $y^3\,x\,x - y^3\,v\,v$ dividi per $x - v$ et dare pro quotiente partes singulares dignitatis inferioris ex $x + v$ ductas in y^3. similiter $y^3\,v\,v - e^3\,v\,v$ dividitur per $y - e$ et dat partes singulares gradus inferioris y vel e, ex binomio $y + e$ ac, pari modo de alijs $y^3\,x\,x - e^3\,x\,x$ et $e^3\,x\,x - e^3\,v\,v$ ratiocinandum esse evidens est.

. . . . . [Exemple] . . . . . .

« Notandum est autem nos semper supposuisse tangentem duci versus y, cum tamen accidere possit ut a parte contraria ducenda sit vel etiam ut sit parallela y. Parallelismus agnoscitur, cum ex datis omnes quantitates affirmatae in quibus a, sunt aequales negatis : tunc enim a est infinitae longitudinis, et consequenter tangens est ipsi parallela.

« Cum vero quantitates affirmatae sunt maiores negatis, tangens ducitur versus y, ut explicuimus.

« At si sunt minores, mutanda sunt omnia signa earundem quantitatum in quibus a reperitur, et eaedem sic mutatae aequandae sunt reliquis in quibus x. Ac tunc constructa aequatione, a sumenda est a parte contratria. Cuius ratio est quod eo casu e sit maior y, et applicatio fieri debeat non ad $y - e$, sed ad $e - y$, quo idcirco signa mutantur, ut norunt analystae.

. . . . . [Exemple] . . . . . .

« Hac regula facile venire possumus in notitiam num curvae flectantur necnon (?), et puncta flexus, vel (?) etiam maximam et minimam applicatarum, siquidem habeant, determinare.

« Exemplum damus in conchoide nova * cuius descriptio est Miscellan: cap: 5....

. . . . . .

« Et haec quidem Regula ducendi tangentes generalis est : ceterum in curvis infinitis quae sub unam descriptionem cadunt ex generali regula deduci possunt compendia quae Laborem calculi minuunt.....

. . . . . [Exemple des « courbes en perle »]. . . . . .

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* Il s'agit de la « conchoide de SLUSE » ; cf. p. ex. G. LORIA, *Spezielle ebene Kurven*, Leipzig, 1910, Bd. 1, S. 74.

The Foundation of Electromagnetism (1820)

HANS CHRISTIAN OERSTED was born in Rudkjoebing in the isle of Langeland, Denmark, on August 14, 1777; he was professor at the University of Copenhagen and died there on March 9, 1851. The experiments which he began in April 1820 are among the most memorable in the whole history of science. They enabled him to correlate two departments of natural philosophy, hitherto separate, — electricity and magnetism, — and thus to carry one step further the unification of knowledge. The pregnancy of OERSTED's discovery is so obvious that additional remarks are unnecessary. For the convenience of scholars teaching the history of physics and of their students, we publish facsimiles of the original Latin paper issued independently by OERSTED on July 21, 1820, and of the English translation transmitted by himself at the same time to the *Annals of Philosophy* (16. 273-276, London 1820).

The portrait illustrating our facsimile and which we owe to the courtesy of Dr. J. W. S. JOHNSSON of Copenhagen reproduces a lithography made by the Danish portraitist EMILIUS DITLEV BAERENTZEN (1799-1868). BEARENTZEN made portraits of almost all the Danes of his time who achieved greatness ; he used to draw them directly on the stone.

GEORGE SARTON.

H.C. Örsted.

EXPERIMENTA

CIRCA EFFECTUM

CONFLICTUS ELECTRICI IN ACUM MAGNETICAM.

Prima experimenta circa rem, quam illustrare aggredior, in scholis de Electricitate, Galvanismo et Magnetismo proxime-superiori hieme a me habitis instituta sunt. His experimentis monstrari videbatur, acum magneticam ope apparatus galvanici e situ moveri; idque circulo galvanico cluso, non aperto, ut frustra tentaverunt aliquot abhinc annis physici quidam celeberrimi. Cum autem hæc experimenta apparatu minus efficaci instituta essent, ideoque phænomena edita pro rei gravitate non satis luculenta viderentur, socium adscivi amicum Esmarch, regi a consiliis justitiæ, ut experimenta cum magno apparatu galvanico, a nobis conjunctim instructo, repeterentur et augerentur. Etiam vir egregius Wleugel, eques auratus ord. Dan. et apud nos præfectus rei gubernatoriæ, experimentis interfuit, nobis socius et testis. Præterea testes fuerunt horum experimentorum vir excellentissimus et a rege summis honoribus decoratus *Hauch*, cujus in rebus naturalibus scientia jam diu inclaruit, vir acutissimus Reinhardt, Historiæ naturalis Professor, vir in experimentis instituendis sagacissimus Jacobsen, Medicinæ Professor, et Chemicus experientissimus Zeise, Philosophiæ Doctor. Sæpius equidem solus experimenta circa materiam propositam institui, quæ autem ita mihi contigit detegere phænomena, in conventu horum virorum doctissimorum repetivi.

In experimentis recensendis omnia præteribo, quæ ad rationem rei inveniendam quidem conduxerunt, hac autem inventa rem amplius illustrare nequeunt; in eis igitur, quæ rei rationem perspicue demonstrant, acquiescamus.

Apparatus galvanicus, quo usus summus, constat viginti receptaculis cupreis rectangularibus, quorum et longitudo et altitudo duodecim æqualiter est pollicum, latitudo autem duos pollices et dimidium vix excedit. Qvodvis receptaculum duabus laminis cupreis instructum est ita inclinatis, ut baculum cupreum, qui laminam zinceam in aqua receptaculi proximi sustentat, portare possint. Aqua receptaculorum $\frac{1}{20}$ sui ponderis acidi sulphurici et pariter $\frac{1}{60}$ acidi nitrici continet. Pars cujusque laminæ Zinceæ in aqua submersa Qvadratum est, cujus latus circiter longitudinem 10 pollicum habet. Etiam apparatus minores adhiberi possunt, si modo filum metallicum candefacere valeant.

Conjungantur termini oppositi apparatus galvanici per filum metallicum, quod brevitatis causa in posterum conductorem conjungentem vel etiam filum conjungens appellabimus. Effectui autem, qui in hoc conductore et in spatio circumjacente locum habet, conflictus electrici nomen tribuemus.

Ponatur pars rectilinea hujus fili in situ horizontali super acum magneticam rite suspensam, eique parallela. Si opus fuerit, filum conjungens ita flecti potest, ut pars eius idonea situm ad experimentum necessarium obtineat. His ita comparatis, acus magnetica movebitur, et quidem sub ea fili conjungentis parte, quæ electricitatem proxime a termino negativo apparatus galvanici accipit, occidentem versus declinabit.

Si distantia fili conjungentis ab acu magnetica $\frac{3}{4}$ pollices non excedit, declinatio acus angulum circiter 45° efficit. Si distantia augetur, anguli decrescunt ut crescunt distantiæ. Cæterum declinatio pro efficacia apparatus varia est.

Filum conjungens locum mutare potest vel orientem vel occidentem versus, dummodo situm acui parallelum teneat, sine alia effectus mutatione, quam respectu magnitudinis; itaque effectus attractioni minime tribui potest, nam idem acus magneticæ polus, qui ad filum conjungens accedit, dum ei ad latus orientale positum est, ab eadem recedere deberet, quando locum ad latus occidentale occupat, si hæ declinationes ab attractionibus vel repulsionibus penderent. Conductor conjungens e pluribus filis aut tæniis metallicis connexis constare potest. Natura metalli effectus non mutat, nisi forte quoad qvantitatem. Fila ex platino, auro, argento, orichalco, ferro, tænias e plumbo et stanno, massam hydrargyri æquali cum successu adhibuimus. Conductor aqua interrupta non omni effectu caret, nisi interruptio spatium plurium pollicum longitudinis complectatur.

Effectus fili conjungentis in acum magneticam per vitrum, per metalla, per lignum, per aquam, per resinam, per vasa figlina, per lapides transeunt; nam interjecta tabula vitrea metallica vel lignea minime tolluntur, nec tabulis ex vitro, metallo et ligno simul interjectis evanescunt, imo vix decrescere videntur. Idem est eventus, si interjicitur discus electrophori, tabula ex porphyrita, vas figlinum, si vel aqua repletum sit. Experimenta nostra etiam docuerunt, effectus jam memoratos non mutari, si acus magnetica pyxide ex orichalco aqua repleta includitur. Effectuum transitum per omnes has materias in electricitate et galvanismo antea nunquam observatum fuisse, monere haud opus est. Effectus igitur. qui locum habent in conflictu electrico, ab effectibus unius vel alterius vis electricæ quam maxime sunt diversi.

Si filum conjungens in plano horizontali sub acu magnetica ponitur, omnes effectus idem sunt ac in plano super acum, tantummodo in directione inversa. Acus enim magneticæ polus, sub quo ea est fili conjungentis pars, quæ electricitatem proxime a termino negativo apparatus galvanici accipit, orientem versus declinabit.

Ut facilius hæc memoria retineantur, hac formula utamur: Polus super quem intrat electricitas negativa ad occidentem, infra quem ad orientem vertitur.

Si filum conjungens in plano horizontali ita vertitur, ut cum meridiano magnetico angulum sensim sensimque crescentem formet, declinatio acus magneticæ augetur, si motus fili tendit versus locum acus deturbatæ; sed minuitur, si filum ab hoc loco discedit.

Filum conjungens in plano horizontali, in quo movetur acus magnetica, ope sacomatis æquilibrata, situm, et acui parallelum, eandem nec orientem nec occidentem versus deturbat, sed tantummodo in plano inclinationis nutare facit, ita ut polus, penes quem ingreditur in filum vis negative electrica deprimatur, quando ad latus occidentale, et elevetur, quando ad orientale situm est.

Si filum conjungens perpendiculare ad planum meridiani magnetici, vel supra vel infra acum ponitur, hæc in quiete permanet; excepto si filum sit polo admodum propinquum: tum enim elevatur polus, quando introitus fit a parte occidentali fili, et deprimitur quando ab orientali fit.

Quando filum conjungens perpendiculare ponitur e regione polo acus magneticæ, et extremitas superior fili electricitatem a termino negativo apparatus galvanici accipit, polus orientem versus movetur; posito autem filo e regione puncto inter polum et medium acus sito, occidentem versus agitur. Quando extremitas fili superior electricitatem a termino positivo accipit, phænomena inversa occurrunt.

Si filum conjungens ita flectitur, ut ad ambas flexuræ partes sibi fiat parallelum, aut duo formet crura parallela, polos magneticos pro diversis rei conditionibus repellit aut attrahit. Ponatur filum e regione polo alteriutri acus, ita ut planum crurum parallelorum sit ad meridianum magneticum perpendiculare, et conjugatur crus orientale cum termino negativo, occidentale cum positivo apparatus galvanici; quibus ita instructis, polus proximus repelletur, vel ad orientem vel ad occidentem pro situ plani crurum. Conjuncto crure orientali cum termino positivo et occidentali cum termino negativo, polus proximus attrahitur. Quando planum crurum ponitur perpendiculare ad locum inter polum et medium acus, iidem, tantummodo inversi, occurrunt effectus.

Acus ex orichalco, ad instar acus magnetieæ suspensa, effectu fili conjungentis non movetur. Etiam acus ex vitro, vel ex sic dicto gummi lacca, simili experimento subjectæ in quiete manent.

Ex his omnibus momenta quædam ad rationem horum phænomenorum reddendam afferre liceat.

Conflictus electricus non nisi in particulas magneticas materiæ agere valet. Videntur omnia corpora non-magnetica per conflictum electricum penetrabilia esse;

magnetica vero, aut potius particulæ eorum magneticæ transitui hujus conflictus resistere, quo fit, ut impetu virium certantium moveri possint.

Conflictum electricum in conductore non includi, sed, ut jam diximus, simul in spatio circumjacente idque satis late dispergi, ex observationibus jam propositis satis patet.

Similiter ex observatis colligere licet, hunc conflictum gyros peragere, nam hæc esse videtur conditio, sine qua fieri nequeat, ut eadem pars fili conjungentis, quæ infra polum magneticum posita eum orientem versus ferat, supra posita eundem occidentem versus agat; hæc enim gyri est natura, ut motus in partibus oppositis oppositam habeant directionem. Præterea motus per gyros cum motu progressivo, juxta longitudinem conductoris, conjunctus, cochleam vel lineam spiralem formare debere videtur, quod tamen, nisi fallor, ad phænomena hucusque observata explicanda nihil confert.

Omnes in polum septentrionalem effectus, hic expositi, facile intelliguntur, ponendo, vim vel materiam negative electricam lineam spiralem dextrorsum flexam percurrere, et polum septentrionalem propellere, in meridionalem autem minime agere. Effectus in polum meridionalem similiter explicantur, si vi vel materiæ positive electricæ motum contrarium et facultatem in polum meridionalem non autem in septentrionalem agendi tribuimus. Hujus legis cum natura congruentia melius repetitione experimentorum quam longa explicatione perspicietur. Dijudicatio autem experimentorum multo fiet facilior, si cursus virium electricarum in filo conjungente signis pictis vel incisis indicatus fuerit.

Dictis hoc tantum adjiciam: Demonstrasse me in libro septem abhinc annis edito, calorem et lucem esse conflictum electricum. Ex observationibus nuper adlatis jam concludere licet, motus per gyros etiam in his effectibus occurrere; quod ad phænomena, quæ polaritatem lucis appellant, illustranda perquam facere puto.

Dabam Hafniæ d. 21de Julii 1820.

Johannis Christianus Örsted.

Eques auratus Ordinis Dannebrogici, in Universitate Hafniensi Prof. Physices Ord., Secretarius Societatis Regiæ Scientiarum Hafniensis.

Typis *SCHULTZIANIS.*

(ANNALS OF PHILOSOPHY. VOL. XVI. P. 273—76. LONDON 1820)

*Experiments on the Effect of a Current of Electricity on the Magnetic Needle.** By John Christian Oersted, Knight of the Order of Danneborg, Professor of Natural Philosophy, and Secretary to the Royal Society of Copenhagen.

The first experiments respecting the subject which I mean at present to explain, were made by me last winter, while lecturing on electricity, galvanism, and magnetism, in the University. It seemed demonstrated by these experiments that the magnetic needle was moved from its position by the galvanic apparatus, but that the galvanic circle must be complete, and not open, which last method was tried in vain some years ago by very celebrated philosophers. But as these experiments were made with a feeble apparatus, and were not, therefore, sufficiently conclusive, considering the importance of the subject, I associated myself with my friend Esmarck to repeat and extend them by means of a very powerful galvanic battery, provided by us in common. Mr. Wleugel, a Knight of the Order of Danneborg, and at the head of the Pilots, was present at, and assisted in, the experiments. There were present likewise Mr. Hauch, a man very well skilled in the Natural Sciences, Mr. Reinhardt,

* Translated from a printed account drawn up in Latin by the author, and transmitted by him to the Editor of the *Annals of Philosophy*.

Professor of Natural History, Mr. Jacobsen, Professor of Medicine, and that very skilful chemist, Mr. Zeise, Doctor of Philosophy. I had often made experiments by myself; but every fact which I had observed was repeated in the presence of these gentlemen.

The galvanic apparatus which we employed consists of 20 copper troughs, the length and height of each of which was 12 inches; but the breadth scarcely exceeded 2¼ inches. Every trough is supplied with two plates of copper, so bent that they could carry a copper rod, which supports the zinc plate in the water of the next trough. The water of the troughs contained $\frac{1}{15}$th of its weight of sulphuric acid, and an equal quantity of nitric acid. The portion of each zinc plate sunk in the water is a square whose side is about 10 inches in length. A smaller apparatus will answer provided it be strong enough to heat a metallic wire red hot.

The opposite ends of the galvanic battery were joined by a metallic wire, which, for shortness sake, we shall call the *uniting conductor*, or the *uniting wire*. To the effect which takes place in this conductor and in the surrounding space, we shall give the name of the *conflict of electricity*.

Let the straight part of this wire be placed horizontally above the magnetic needle, properly suspended, and parallel to it. If necessary, the uniting wire is bent so as to assume a proper position for the experiment. Things being in this state, the needle will be moved, and the end of it next the negative side of the battery will go westward.

If the distance of the uniting wire does not exceed three-quarters of an inch from the needle, the declination of the needle makes an angle of about 45°. If the distance is increased, the angle diminishes proportionally. The declination likewise varies with the power of the battery.

The uniting wire may change its place, either towards the east or west, provided it continue parallel to the needle, without any other change of the effect than in respect to its quantity. Hence the effect cannot be ascribed to attraction; for the same pole of the magnetic needle, which approaches the uniting wire, while placed on its east side, ought to recede from it when on the west side, if these declinations depended on attractions and repulsions. The uniting conductor may consist of several wires, or metallic ribbons, connected together. The nature of the metal does not alter the effect, but merely the quantity. Wires of platinum, gold, silver, brass, iron, ribbons of lead and tin, a mass of mercury, were employed with equal success. The conductor does not lose its effect, though interrupted by water, unless the interruption amounts to several inches in length.

The effect of the uniting wire passes to the needle through glass, metals, wood, water, resin, stoneware, stones; for it is not taken away by interposing plates of glass, metal or wood.

Even glass, metal, and wood, interposed at once, do not destroy, and indeed scarcely diminish the effect. The disc of the electrophorus, plates of porphyry, a stone-ware vessel, even filled with water, were interposed with the same result. We found the effects unchanged when the needle was included in a brass box filled with water. It is needless to observe that the transmission of effects through all these matters has never before been observed in electricity and galvanism. The effects, therefore, which take place in the conflict of electricity are very different from the effects of either of the electricities.

If the uniting wire be placed in a horizontal plane under the magnetic needle, all the effects are the same as when it is above the needle, only they are in an opposite direction; for the pole of the magnetic needle next the negative end of the battery declines to the east.

That these facts may be the more easily retained, we may use this formula—the pole *above* which the *negative* electricity enters is turned to the *west*; *under* which, to the *east*.

If the uniting wire is so turned in a horizontal plane as to form a gradually increasing angle with the magnetic meridian, the declination of the needle *increases*, if the motion of the wire is towards the place of the disturbed needle; but it *diminishes* if the wire moves further from that place.

When the uniting wire is situated in the same horizontal plane in which the needle moves by means of the counterpoise, and parallel to it, no declination is produced either to the east or west; but an *inclination* takes place, so that the pole, next which the negative electricity enters the wire, is *depressed* when the wire is situated on the *west* side, and *elevated* when situated on the *east* side.

If the uniting wire be placed perpendicularly to the plane of the magnetic meridian, whether above or below it, the needle remains at rest, unless it be very near the pole; in that case the pole is *elevated* when the entrance is from the *west* side of the wire, and *depressed*, when from the *east* side.

When the uniting wire is placed perpendicularly opposite to the pole of the magnetic needle, and the upper extremity of the wire receives the negative electricity, the pole is moved towards the east; but when the wire is opposite to a point between the pole and the middle of the needle, the pole is most towards the west. When the upper end of the wire receives positive electricity, the phenomena are reversed.

If the uniting wire is bent so as to form two legs parallel to each other, it repels or attracts the magnetic poles according to the different conditions of the case. Suppose the wire placed opposite to either pole of the needle, so that the plane of the parallel legs is perpendicular to the magnetic meridian, and let the eastern leg be united with the negative end, the western leg with the positive end of the battery: in that case the nearest

pole will be repelled either to the east or west, according to the position of the plane of the legs. The eastmost leg being united with the positive, and the westmost with the negative side of the battery, the nearest pole will be attracted. When the plane of the legs is placed perpendicular to the place between the pole and the middle of the needle, the same effects recur, but reversed.

A brass needle, suspended like a magnetic needle, is not moved by the effect of the uniting wire. Likewise needles of glass and of gum lac remain unacted on.

We may now make a few observations towards explaining these phenomena.

The electric conflict acts only on the magnetic particles of matter. All non-magnetic bodies appear penetrable by the electric conflict, while magnetic bodies, or rather their magnetic particles, resist the passage of this conflict. Hence they can be moved by the impetus of the contending powers.

It is sufficiently evident from the preceding facts that the electric conflict is not confined to the conductor, but dispersed pretty widely in the circumjacent space.

From the preceding facts we may likewise collect that this conflict performs circles; for without this condition, it seems impossible that the one part of the uniting wire, when placed below the magnetic pole, should drive it towards the east, and when placed above it towards the west; for it is the nature of a circle that the motions in opposite parts should have an opposite direction. Besides, a motion in circles, joined with a progressive motion, according to the length of the conductor, ought to form a conchoidal or spiral line; but this, unless I am mistaken, contributes nothing to explain the phenomena hitherto observed.

All the effects on the north pole above-mentioned are easily understood by supposing that negative electricity moves in a spiral line bent towards the right, and propels the north pole, but does not act on the south pole. The effects on the south pole are explained in a similar manner, if we ascribe to positive electricity a contrary motion and power of acting on the south pole, but not upon the north. The agreement of this law with nature will be better seen by a repetition of the experiments than by a long explanation. The mode of judging of the experiments will be much facilitated if the course of the electricities in the uniting wire be pointed out by marks or figures.

I shall merely add to the above that I have demonstrated in a book published five years ago that heat and light consist of the conflict of the electricities. From the observations now stated, we may conclude that a circular motion likewise occurs in these effects. This I think will contribute very much to illustrate the phenomena to which the appellation of polarization of light has been given.

Copenhagen, July 21, 1820. JOHN CHRISTIAN OERSTED.

The Atomic View of Matter in the XVth, XVIth, and XVIIth Centuries.

§ 1. *Introduction.*

It is generally conceived that atomic philosophy was in abeyance, from the time when the authority of ARISTOTLE displaced it, until the days of DALTON. Those who have looked further into the matter are aware of the atomic leanings of GASSENDI about the middle of the seventeenth century, and those of BOYLE and NEWTON which quickly followed. It is, however, seldom realised that there was a continuous chain of atomic thought from the fifteenth century to the time of DALTON. Our purpose here is to trace this thought as far as BOYLE in the works of some of its more striking exponents, not neglecting some of its opponents. The consideration of BOYLE, of NEWTON and of their successors in the eighteenth century demands separate treatment.

The current attitude may be illustrated by a statement made by NERNST that DALTON's Atomic Theory « arose, by one effort of modern science, like a phoenix from the ashes of the old Greek philosophy. » (1)

§ 2. *Atomism in the middle ages. The Recovery of Lucretius.*

An examination of the works of many of the Church Fathers and early Christian writers reveals an undercurrent of atomistic tradition. (2) The idea is still there, though it is frequently misunderstood. Thus, ISIDORE OF SEVILLE (560-636), (3) the Venerable

(1) W. NERNST, *Theoretical Chemistry*, English translation of 10th German edition, London 1923.

(2) J. PHILIPPE, *Lucrèce dans la théologie chrétienne de III^e et XIII^e siècle*, Paris, 1896.

(3) ISIDORE OF SEVILLE, *Etymologia*, Lib. XIII.1.11.

BEDE, (672-735), (4) and HRABANUS MAURUS (776-856), (5) all speak of the « atom », referring to discontinuity in bodies, in time and number. WILLIAM OF CONCHES (1080-1154) (6) and VINCENT OF BEAUVAIS (d. about 1268), (7) two of the most distinguished mediaeval scholars, display some knowledge of atomic philosophy, and the former openly taught the atomistic doctrine of DEMOCRITUS and EPICURUS. It may be asked how this knowledge came to mediaeval thinkers. The matter is explained firstly by their copying from each other, and secondly by the fact that a few copies of LUCRETIUS were scattered among the libraries of various churches during the Middle Ages. Thus the 9th century MS. of LUCRETIUS now at Leyden was formerly at St. Martin's at Mainz, the see of HRABANUS. A 10th century MS. also at Leyden was once in the abbey of St. Bertin, near St. Omer. There was a copy at Corbie, near Amiens, about 1200, which has since been lost. Other copies certainly existed. Fragments of some which have survived are now at Copenhagen and Vienna.

POGGIO BRACCIOLINI (1380-1459), one of the early promoters of classical literature in Italy, undertook the task of searching the monasteries for ancient manuscripts, and discovered, among other works, a copy of LUCRETIUS. In 1414 he brought it to Italy from Germany. This MS. of POGGIO is now lost, but a copy of it survives. From such a copy was prepared the first printed edition, which appeared in Brescia in 1473. It was reprinted in Verona in 1486, and attracted much attention.

§ 3. *Nicholas of Cusa.*

But even prior to the appearance in print of the work of LUCRETIUS there was some revival of atomic thought. Notably NICHOLAS OF CUSA (1401-1464) recalled the basic ideas of the atomists from the theoretical point of view.

NICHOLAS OF CUSA, whose real name was KHRYPFFS, CHRYFFTZ or KREBS, was the son of a fisherman and was born at Cues, on the

(4) BEDE, *Opera*, Cologne 1688, Vol. i. page 90.

(5) HRABANUS MAURUS, *Opera a Jac. Pamelio collecta*, Cologne 1626, Vol. I, page 145, Book IX, Chap. i.

(6) *Elementorum philosophiæ libri IV* bound up in the Cologne (1688) edition of BEDE's works. Vol. II, pages 206-230.

(7) VINCENT OF BEAUVAIS, *Speculum major*, Venice 1591, Chap. 2. Book 2,p. 14.

Moselle not far from Trèves. He ran away from home while a boy to escape ill-treatment, and took refuge in the service of a neighbouring nobleman. This man saw in him evidence of exceptional ability, and sent him to study at the school of the « Brothers of the Common Life » at Deventer. NICHOLAS afterwards went to the University of Padua, where he took a degree in law in his twenty-third year. He became one of the most distinguished intellects of his time and takes an important place in the history of thought. In philosophy he was one of those who broke with the scholastic system while it was still orthodox. In mathematical and physical science, he was much in advance of his age, and he was certainly a pioneer of the experimental method. He was one of the earliest thinkers to develop a system of religious toleration. He can be said to have placed himself at the head of nearly all kinds of progressive movements in the Church, in Science, in Letters and in Philosophy. He was in sympathy also with the Humanism.

At Deventer, NICHOLAS had learned to know and love the Latin classics. At Padua, he found himself in the company of the keenest intellectuals, and he became secretary to Cardinal ORSINI, one of the learned princes of the Church, who showed a love for ancient literature. NICHOLAS himself became a keen and successful seeker for ancient MSS., and his library, which still exists, almost intact, is of the greatest interest. With such tastes it was natural that NICHOLAS should meet POGGIO, and the discovery of the work of LUCRETIUS would thus be known to him.

With a knowledge of LUCRETIUS, it is easy to understand why NICHOLAS introduces in his *De Mente Idiotae* the question : « What dost thou understand by an atom? » (8). To that question, he replies : « Under mental consideration that which is continuous becomes divided into the ever divisible, and the multitude of parts progresses to infinity. But by actual division we arrive at an actually indivisible part which I call an atom. For an atom is a quantity, which on account of its smallness is actually indivisible. »

NICHOLAS was, moreover, but one of a school of Renaissance thinkers who were influenced by LUCRETIUS. Of these, FRACASTORO is the most prominent.

(8) NICOLAS OF CUSA, *Opera*, Basel 1555, «De Mente Idiotae», Book III, Chap. 9, p. 162.

§ 4. *Girolamo Fracastòro.*

Very soon after the first printing of the work of LUCRETIUS there was born GIROLAMO FRACASTORO, a man who was deeply stirred by the awakening of atomistic ideas. He was born in Verona in 1478 and died near that city in 1553. He came of a stock which had produced many distinguished physicians, and he received the most complete education available in his day. He attended the University of Padua, where he occupied himself with medicine, mathematics and philosophy. FRACASTORO thus associated with many brilliant men, and there is no doubt that several of them exercised great influence upon him. Among them were GIAMBATTISTA RHAMNUSIO (1485-1587), the Italian HAKLUYT, who inscribed to FRACASTORO his *Viaggi e Navigazioni*, and ANDREA COTTA and ANDREA NAVAGERO (1483-1529), two excellent Latin poets. The latter was well versed in Greek and Latin Literature and edited for the Aldine press the works of QUINTILIAN, VIRGIL, OVID, TERENCE, HORACE, the Speeches of CICERO, as well as LUCRETIUS. With such companions FRACASTORO early developed facility as a writer of elegant verse. The atomic theory of LUCRETIUS was quite familiar to him, and the conception of the « seeds of disease » or *seminaria hypothesis* which has become attached to his name has its basis in LUCRETIUS. In the *De rèrum natura* of LUCRETIUS are, in fact, actual references to seeds that are helpful to life and seeds which cause disease and death. (9)

FRACASTORO in his *De Contagionibus* (10) in dealing with infection by means of fomites (11) supposes the existence of minute invisible particles which convey the infection. These can lurk in the recesses of a porous substance. To them he applies the term *semina*. When dealing with infection at a distance (12) he elaborates a suggestion as to how the seeds of contagion may be carried to a distance and on to the world at large — the idea being that all bodies, especially moist and volatile ones, constantly give off small particles, which may be perceived as vapours or odours.

In his *De sympathia et antipathia* FRACASTORO actually puts

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(9) LUCRETIUS, *De Rerum natura*, Book VI, pages 257 and 272. Clarendon Press. (Trans. BAILEY).

(10) HIERONYMO FRACASTORO, *Opera Omnia*, Venice 1555, page 105, *et seq.*

(11) *Op. cit. De Contagione*, Book 1, Chap. 4, page 106 B.

(12) *Op. cit. De Contagione*, Book 1, Chap. 7, page 108 C.

forward a corpuscular point of view to explain actions of a chemical order, (13) and he expresses assent to the atomic theory of EPICURUS, DEMOCRITUS and LUCRETIUS.

§ 5. *Ramus.*

In the second half of the 16th century began a definite agitation against the old Aristotelian physics. The most vigorous opponent of ARISTOTLE was perhaps PETER RAMUS (PIERRE DE LA RAMÉE, 1515-1572), one of the most learned humanists of the time. He was born at Cuth, near Soissons in Picardy, and went to Paris about 1523, when but 8 years old, and became a lackey in the Collège de Navarre.

RAMUS early developed the strongest inclination for learning. Soon he broke with scholastic Aristotelianism, pushing his opposition to revolutionary extremes. Without discretion or restraint, he attacked the great idol, and when scarcely 21 he presented a thesis, the subject of which was the audacious proposition that « all that ARISTOTLE has said is false ». His disputants being unable to appeal to the authority of ARISTOTLE without begging the question, were unable to make any headway. As a result, after assailing his thesis for a whole day and having their arguments refuted with great spirit, subtlety and directness, they were at length obliged to admit him to the degree with honours. Such was the way in which academic distinction was earned in those days !

RAMUS dreamt of compassing a thorough regeneration of dialectic. He expounded his plans in 1543 in his *Dialecticae Institutiones* and *Animadversiones in Dialecticam Aristotelis*. The works provoked violent opposition ; their publication was prohibited and they were ordered to be burned. Their author was silenced by order of FRANCIS I. On the death of FRANCIS in 1544, RAMUS resumed the teaching of philosophy at Presles, and re-edited his works. In 1551 he was appointed regius professor of rhetoric and philosophy at Paris. His spirit of inquiry ultimately led him to embrace the Reformed faith and he was obliged to flee Paris. His house was pillaged and his library burned. In 1563 he was restored to his chair, but in 1568 affairs were again so threatening

(13) *Op. cit. De sympathia et antipathia*, Book 1, Chap. 5, page 82 B. C.

that he found it advisable to ask permission to travel. Returning to France, he fell a victim to his opponents in the massacre of St. Bartholomew at Paris in 1572.

§ 6. *Giordano Bruno.*

Some twenty years later, in 1583, GIORDANO BRUNO (1548-1600), now 35 years of age and long in rebellion against the scholastic system, presented himself in Paris as an antagonist of the Aristotelian philosophy. BRUNO made a definite stand for a finite state of matter. His atomism or doctrine of ***monads*** is put forward in his difficult poem *De Triplici Minimo et Mensura* published at Frankfurt in 1591, though an idea as to ultimate particles is seen in his *Acrotismus* of 1588. In that work he says, « The division of natural things has a limit; an indivisible something exists. The division of natural things attains the smallest and last parts which are not perceptible by the aid of human instruments. » (14)

His *De Triplici Minimo* is both metaphysical and geometrical, and in it geometrical considerations and metaphysical explanations are almost inextricably mingled. In the second chapter, BRUNO puts forward his doctrine of « the Minimum » which, being the indivisible unit, is not only the element from which all is made up, but is also the principle, the germ of all existence. (15) The views of BRUNO do not submit of ready analysis, but we may say that the title of the book is taken from his view that there are three minima : (16)

(*a*) A general metaphysical minimum or monad, which means primarily the unit at the base of all existence, and secondarily the limit at the base of number — the minimum as the principle of quantity.

(*b*) A physical minimum or atom — the minimum as the principle of the size of bodies.

(*c*) The geometric minimum or point.

BRUNO declares his belief in discontinuity, holding that there is a minimum that is the basis of all things. « Thus the minimum

(14) BRUNO, *Camoeracensis acrotismus seu rationes articulorum physicorum*. Wittenberg 1588. Book VI, Art. 42.

(15) BRUNO, *De triplici minimo*, Frankfurt 1591, Page 9, line 5.

(16) *Op. cit.* page 10.

is found in all things, and if it were not underlying, things would be aught. Without the monad, there would be no number, for it constitutes the species, which determines each genus. It is indeed the ultimate foundation in all things. It is God and fruitful nature, and art interprets it as that which persists beyond, and wich is common to, every genus » (17) « The minimum remains as a constant in all things being above the confines of finiteness : it attains to infinity, causing, uniting, renewing and propagating eternally all creatures, both composite and simple. » (18)

BRUNO reduces all knowledge of nature to a knowledge of the « minimum », and he concludes that « if contemplation pursues the form of nature, it ought to begin at the minimum, it ought to consist in the examination of the minimum, and it ought to stop with the apprehension of the minimum ». (19) He deduces that the knowledge of the minimum is absolutely necessary as a foundation for natural science, mathematics and metaphysics. (20) He holds that without this principle neither physicist nor mathematician nor philosopher could work. (21) He seeks to refute current proofs of divisibility to infinity, and states the origin and base of all errors in physics as in mathematics to be the conception of a division of a continuous whole to infinity. Thus all error can be reduced to ignorance of the minimum. (22)

BRUNO has a lengthy argument in opposition to ARISTOTLE's conception of continuity, (23) but although he postulates empty space and atoms, 'as do LEUCIPPUS and DEMOCRITUS,' his atomism differs from their materialistic philosophy which considered life and soul as products of union of the atoms. The minimum of BRUNO is the « primodial force », the creating germ and the divine spark by which all things exist. Each kind of substance must possess a definite minimum from which it is established and into which it could be reduced. (24)

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(17) *Op. cit.* page 9, lines 11-17.
(18) *Op. cit.* page 9, lines 18-22.
(19) *Op. cit.* page 18, note 79.
(20) *Op. cit.* page 20, note 1.
(21) *Op. cit.* page 16, lines 79-82.
(22) *Op. cit.* page 23.
(23) *Op. cit.* pages 24-31.
(24) *Op. cit.* Book IV, c. 2. Heading and notes on p. 102.

§ 7. *Bacon and Descartes.*

BRUNO sojourned for a time in England where a contemporary opponent of Aristotelianism was FRANCIS BACON (1561-1626).

To BACON, all that we now call the natural sciences was the soundest philosophy, and while he bitterly attacked the principles and practice of the Peripatetics, (25) and deplored the condition to which natural philosophy had fallen owing to the want of proper appreciation, (26) he promulgated a new method by which all knowledge should be collected and arranged. The evidence of the senses was to form the starting point, and experiments were to be performed to check any information obtained. « Axioms » were to be framed as a result. These were to be examined for the discovery of additional implied facts, and the existence of such facts was to be determined by further experiments.

In considering the ultimate constitution of matter, he was evidently attracted by the atomic doctrine of DEMOCRITUS (27) but he considered the hypothesis of a vacuum and that of the unchangeableness of matter as false assumptions, while the actually existing ultimate particles as conceived by him were not atoms proper but were always capable of further division. (28) His idea was that there was only one kind of primitive matter capable of transmutation into the different kinds that may be perceived. (29)

In his attempt to propound a new method, he was joined by by RENÉ DESCARTES (1596-1650) whose standpoint, however, was fundamentally different. The object of DESCARTES was to found a system which should be solid, clear and convincing. It was to be based on first principles whose validity was beyond doubt, and from these everything was to be explained deductively. The validity of any inference was tested by its approximation to a mathematical demonstration. It was his ambition to apply the geometrical method to universal science and to make it the method of metaphysics. (30)

In his search for his clear first principles, he attempted to divest

(25) BACON, *Novum Organum*, London 1620, I. 63.
(26) BACON, *The Great Instauration, Plan of the work.*
(27) BACON, *De Principiis atque Originibus.*
(28) BACON, *Novum Oragnum*, II, 9 and 48.
(29) BACON, *Thoughts on the nature of things*, II.
(30) DESCARTES, *Discourse on Method*, (Ed. EVERYMAN's Lib.), pages 7 and 16.

himself of every preconceived notion, and as a result he concluded that the most clearly apprehended of all truths was summed up in his famous dictum, *Cogito, ergo sum*. Taking this as a basis and applying his method, DESCARTES was led to the conception of the evolution of the material universe and to his view of the ultimate constitution of matter. His systematic work, *Principia Philosophiae* (1644), treats in successive sections of the principles of human knowledge, of the principles of material things, of the visible world and of the earth, and after recapitulating the principles laid down in the *Meditationes*, his philosophical system — and especially his natural philosophy — is developed. The aim of his natural philosophy is to give an account of all that can be discovered of nature by thought. He holds that the one essential attribute of bodies is extension (31) in length, breadth and depth, and other attributes and qualities are secondary when considering the nature of matter. (32)

In a further development of his Physics the conclusions that are of importance from the standpoint of the atomists are the following :

(a) He denies the existence of a vacuum ; (33) holding that in the philosophical sense a space, in which there is no substance, cannot exist, as the extension of space is no different from that of a body.

(b) The absence of a vacuum had to be reconciled with the existence of motion, and this leads him to conclude that matter could be infinitely divided. (34)

(c) Allowing no limit to the divisibility of matter, he denies the possibility of the existence of atoms. (35)

DESCARTES is thus far from being an atomist, for although he regards all matter as made up of particles so small as to be imperceptible by our senses, (36) yet these particles are not, in his opinion, indivisible, nor is there any void. (37)

(31) DESCARTES, *Principia Philosophiae*, I, 53, II. 4.
(32) *Op. cit.* II, 4.
(33) *Op. cit.* II. 16.
(34) *Op. cit.* II. 34.
(35) *Op. cit.* II. 20.
(36) *Op. cit.* IV. 201.
(37) *Op. cit.* IV. 202.

§ 8. *Early seventeenth century atomists.* (a) *The Germanic School.*

The influence of DESCARTES during the seventeenth century was enormous. Nevertheless, the atomic view of matter was very far from lacking supporters of a humbler order. An able exponent of the corpuscular theory during this period was DANIEL SENNERT (1572-1637), a native of Breslau, who studied philosophy and medicine in Wittenberg, in Leipzig, Jena and Frankfurt. He became Professor of Medicine at Wittenberg, and as a physician he was one of the first to take up a physical atomistic view. His fame as a practitioner and teacher received universal acknowledgment. He died of the plague in 1637.

SENNERT first shows his adherence to atomism in his *De chymicorum cum Aristotelicis et Galenicis consensu ac dissensu* published at Wittenberg in 1619. In his *Hypomnemata Physica*, (Wittenberg in 1636), SENNERT seeks to set forth atomic doctrine in relation to the old teaching concerning the four elements. He writes that « in considering natural things, subject to generation and corruption, one must necessarily suppose simple bodies of a particular kind, from which the composite bodies arise and into which they are resolved. These simple bodies are physical, not mathematical minima, and are so small that they cannot be perceived by the senses, and these « minima » are the smallest indivisible particles to which all natural bodies owe their existence. » (38) SENNERT holds that there must be atoms of more than one type — atoms of elements and atoms of composite bodies (secondary atoms), (39) and in connection with the different kinds of elements he gives four types of elementary atoms, viz : atoms of fire, atoms of air, atoms of water and atoms of earth.(40) He also supposes that living things, such as plants and animals, are composed of atoms (41) and he illustrates the smallness of atoms by comparing them with the smallest animals, of which he knows « Acari » and « Sirones» the insects of the « Itch » (42)

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(38) D. SENNERT, *Hypomnemata Physica*, Wittenberg 1636, Hypomnema III, Chap. I, p. 90.

(39) *Op. cit.* p. 94.

(40) *Op. cit.* p. 96-97.

(41) *Op. cit.* page 112.

(42) *Op. cit.* page 114.

The observation of these minute creatures frequently awoke atomic thought and language among his contemporaries.

SENNERT points out that chemical operations are capable of explanation when the existence of atoms is supposed, and that a consideration of certain operations shows how small these atoms must be. (43) For example, in evaporation, the vapour from spirits of wine penetrates four thicknesses of writing paper : in distillation, a large volume of vapour containing myriads of atoms yields scarcely a small drop : in melting a mass of gold and silver, the atoms run together and in the resultant mass the different metals cannot be detected, yet each may be obtained again from the mass in its original state.

He further holds that the essential form of a substance is retained in its atomic parts, (44) and quotes the experiment of melting gold and silver so that the atoms of each are thoroughly mixed, and the individual metals cannot be distinguished, though each retains its own form, for on treating the mass with nitric acid the silver is dissolved and the gold remains. Also if mercury be sublimed, dissolved, or suffer other changes by a variation of the atoms into which it is resolved when it becomes mixed with others, it always retains its essential form, and can be readily separated from bodies with which it is mixed, and it may be obtained in its pristine form of running mercury.

In the year following the publication of SENNERT's *De chymicorum consensu ac dissensu* and in the same year as the publication of BACON's *Novum Organum* (1620), a book appeared at Leyden by one DAVID GORLAEUS (van Goorle) entitled *Exercitationes philosophicae quibus universa fere discutitur Philosophia Theoretica*, which is of some interest for our theme. GORLAEUS of Utrecht, in this and in his other works, ranged himself against the folowers of ARISTOTLE, though he approached somewhat the philosophical outlook of DESCARTES. He points out in a chapter dealing with Atoms (45) that if an entity be given and the body has material parts, then these parts must be indivisible, or the body would have no material being when such parts come together. Further, if atoms of a definite size be granted, then all quanti-

(43) *Op. cit.* p. 108.
(44) *Op. cit.* pp. 114 and 115.
(45) D. GORLAEUS, *Exercitationes philosophicae*, Leyden 1620, pages 235-239.

ties are divisible into these atoms, and the end of the division of a body is not « non-existence » but only the separation into other parts. Following this he argues the impossibility of an endless number, and the necessity of the atoms having size and weight. He also discusses the shape of the atoms, and questions whether they are round or cubical, and concludes the section with the statement that « whatever the shape may be, the atoms are so small that they cannot be perceived by the senses and scarcely apprehended by the mind. » (46)

Another Dutch philosopher, JOHANNES PHOCYLIDES HOLWARDA (1618-1651) also takes sides with the atomists, and declares that « atoms are nothing more than the smallest corpuscles possible and incapable of further division ». (47) He considers that their movement proceeds from a Creator, (48) but that their different effects result from their shape and their « sympathy and antipathy » (49). He also accepts the existence of a vacuum. (50)

That a corpuscular theory held some place in the thought of the time is further shown by the use of the term « Atom » by the chemist VAN HELMONT (1577-1644). He often employs the term (51) but he does not seem to mean Atoms in the strict sense, but only minute particles. He speaks of the odours, smells and seeds which rise in the air, and states that « a vapour is a cloud of atoms of water rent asunder from each other ». (52)

In 1647, JOHANNES SPERLING (1603-1658) published his *Institutiones Physicae* at Lubeck. His work is borrowed largely from SENNERT. It does not go as far as MAGNEN as to the « atoms » of living creatures and mixtures being divisible, but considers atoms of fire, air, water, earth, mixtures, plants and animals. He writes that « Atoms are the most minute *effluvia*, particles of the utmost smallness, which may be seen when in groups but are invisible when alone. » (53)

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(46) *Op. cit.* p. 243.

(47) J. P. HOLWARDA, *Philosophia naturalis*, Franeker 1651, Chap. III, p. 6-8.

(48) *Op. cit.* Chap. V, p. 12-14.

(49) *Op. cit.* Chaps. VI and VII, p. 15-20.

(50) *Op. cit.* Chap. XI. p. 27-40.

(51) J. B. VAN HELMONT, *Oriatrike or Physick refined*, London 1662, Chap. 8. para. 15 ; Chap. 13, para 21 & 22 ; Chap. 15, para 27 ; Chap. 11, para 7.

(52) *Op. cit.* Chap 15, para 27.

(53) J. SPERLING, *Institutiones physicae*, Lubeck 1647, p. 787.

In considering the German School of Atomists of this period, mention must be made of JOACHIM JUNGE (1587-1657). He is best known for his botanical work, and he also made some mark as an atomist. According to WOHLWILL, (54) who has had access to all known existing manuscripts relating to JUNGE, his corpuscular ideas were already formed in 1622, and had been built up before he knew SENNERT or BASSO.

In 1629, as Rector and Teacher in a school at Hamburg, he was supposed to teach Aristotelian Physics. Nevertheless, he always pointed out its untenability, and maintained and emphasised the superiority of the corpuscular theory. The notes of his lectures, chiefly of the years 1629-1631, are preserved, and were the basis of a posthumous publication (55). In various discussions he defended the atomic standpoint. On March 23rd, 1633, he opposed « the substantial forms » ; in 1634, he dealt with « chemical elements », and the views he then expressed exhibit remarkable similarity to those later expressed by BOYLE. In this connection, we may note that on May 15th, 1654, HARTLIB sent to BOYLE a « rude draught of JUNGE's philosophy » adding that « as it lyes in a pack bound about with such coarse expressions and terms as he uses, it makes no great show, but if it were fully opened, a great deal would appear to be rich cloth of Arras ».

The detailed treatment of his corpuscular theory is seen in the published accounts of two discussions of March 30th, and April 2nd, 1642. These had very little circulation at the time but they establish the claim of JUNGE as an independent atomist and a scientific ancestor of BOYLE.

§ 9. *Early Seventeenth Century Atomists.* (b) *The Italian School.*

The renaissance of the corpuscular theory was not confined to Germany and Flanders. In Italy GALILEO led the van of the forces arrayed against ARISTOTLE. GALILEO was no atomist but his associate CLAUDE BERIGARD had atomist leanings. He published two works, *Dubitationes in dialogum Galilaei pro terrae immobilitate* (Florence, 1632), and *Circulus Pisanus* (Udine, 1643) in which he comments upon the Physics of ARISTOTLE.

(54) E. WOHLWILL, *Joachim Junguis und die Erneurerung atomischer Lehren in 17 Jahrhundert*, Hamburg 1887.

(55) J. JUNGE, *Doxoscopiae physicae minores* (Ed. M. VOGEL), Hamburg 1662.

BERIGARD was a French physician, who was born at Moulins probably in 1578. He successfully studied literature, philosophy, medicine and mathematics, and after having lived for some time in Paris, went to Pisa in 1628, where he was known to GALILEO. Here he taught philosophy until 1640, when he was invited to occupy the chair in that subject at the University of Padua, where he died in 1664.

His *Circulus Pisanus* is in dialogue form, and in the section headed *In Aristotelis libros de ortu et interitu* he shows a very definite partizanship towards a corpuscular theory, and an antagonism to ARISTOTLE, although no decision is reached in the dialogue. His ideas are rather hazy, but there is approbation of the conception, ascribed to ANAXAGORAS, of the foundation of the universe from an endless number of qualitatively different substances, though he inclines to an atomic composition of these basic substances.

Three years later, 1646, JEAN CHRYSOSTOME MAGNEN (1590-1679) published at Pavia an elaborate work (56) which shows the awakening that had taken place in atomic thought. He commences his *Democritus reviviscens* with a long preface dealing with the life and character of DEMOCRITUS, and then gives a short summary of what he supposes to have been his atomic teaching. He opens by discussing the four elements, (57) which he considers must be reduced to three, viz : earth, water and fire. (58) After this disquisition, he continues his propositions in connection with atoms. (59) He regards the elements as atomic, and he develops the idea « that all things are built up from atoms ». (60) One of his propositions states that « the *continuum* is made up of atoms or an infinite number of corpuscles, adequately distinct amongst themselves and of certain determinate extent ». (61)

In the earlier propositions (62) in which he deals with the atomic composition of things, he reviews reasons why the continuum cannot consist of mathematical points, whether the number

(56) MAGNEN, — *Democritus reviviscens sive de atomis*, Pavia 1646.
(57) *Op. cit.* Prop. IX, page 59.
(58) *Op. cit.* Prop. XI, page 70.
(59) *Op. cit.* « De compositione rerum ex atomis », p. 94.
(60) *Op. cit.* Chap. II, page 104.
(61) *Op. cit.* Prop. XIX, p. 104.
(62) *Op. cit.* Props. XIV-XVIII, pp. 98-103.

be finite, indeterminate or absoluty infinite. In the introduction to Prop. XIX, he states that the atoms are not mathematical points, as these have no dimensions as the atoms have, for the atoms are physically indivisible particles.

He puts forward eight fundamental reasons for the existence of atoms, among which are the following :

(a) Nature shrinks from the endless. (63)

(*b*) « Minima » under different names appear in nearly all Physics. (64)

(*c*) In the building up of bodies, there must be the definite indivisible parts, otherwise one would get matter without form. (65)

(*d*) Decomposition would yield « nothings ». There must be a smallest form. Division does not give an endless number of parts, but quantitative physical parts. (66)

(*e*) The acceptance of an atom rests upon certain chemical experiments, (67) for which he refers to SENNERT's *Hypomnemata Physica III*, chap. 2, and also to some wonderful tales of JACQUES GAFFAREL (1601-1681) as recorded in his *Curiositez inouyes*. (68)

In attempting to define the atom, (69) he states that it is material, simple, purely homogenous, and by its very nature, indivisible. (70) He speaks of atoms of fire, water and earth (71) and says the smallest parts of living creatures and mixtures are not atoms, but are susceptible of division into atoms. In discussing their properties he considers that they have a triple movement,(72) and that « sympathy » exists between atoms. (73) He deals, moreover, with the different conjectures as to the shape of the atoms. (74)

(63) *Op. cit.* p. 105.
(64) *Op. cit.* p. 106.
(65) *Op. cit.* p. 108.
(66) *Op. cit.* p. 109.
(67) *Op. cit.* pp. 109, 110.
(68) J. GAFFAREL, *Curiositez inouyes*, Hamburg, 1629.
(69) *Op. cit.* p. 113. Prop. XX.
(70) *Op. cit.* p. 114.
(71) *Op. cit.* p. 114.
(72) *Op. cit.* Chap. III, p. 116, Prop. XXI.
(73) *Op. cit.* Prop. XXII.
(74) *Op. cit.* Props. XXIII-XXX pp. 122-154.

Another resolute adversary of Aristotelianism in this period was SEBASTIAN BASSO. In his book *Philosophia Naturalis adversus Aristotelem* published at Geneva in 1621, he suggests a return to Atomism. (75) He attempts to give what he regards as the ideas of DEMOCRITUS, ANAXAGORAS, PLATO and EMPEDOCLES with regard to « the Atom ». He points out that these philosophers differed as to the « prima principia » but that all agreed that it was suitable to explain the cause and reason of natural change. The postulate was that all things arose from very small and very different particles, which possessed a different nature in the divided state and also retained it when in combination. BASSO calls the particles « atoms » and holds that they must be understood as having been created by God in the beginning .

§ 10. *Gassendi.*

PIERRE GASSENDI (1592-1655) of Champtercier, near Digne, in Provence, is usually considered as the reviver of Epicureanism. He certainly adapted that system of thought to the exigencies of the philosophy of his time, yet it is clear from what has gone before that Atomism had never ceased to have partisans. GASSENDI is, indeed, only the most prominent member of a flourishing school, which existed before him and persisted after him.

The Epicurean doctrine as understood and modified by GASSENDI is embodied in his *Syntagma philosophiae Epicuri* (Lyons, 1649) a work on which at least twenty years of labour were expended. The *Syntagma* (76) is divided into three parts : Logic, Physics and Ethics. In the first section of his *Physics* he deals with space, time, movement, the *materia prima*, the properties of things, generation and corruption. The question of the existence of a vacuum is fully discussed (77) before mention is made of atoms. The importance which he attaches to the existence of the void is shown by the fact that he gives three long chapters to the demonstration of its necessity.

In considering the Prima Materia (78) his point of view is that there must be something which serves as a concrete principle,

(75) S. BASSO *Philosophia naturalis*, Geneva 1621, Arts. 4 and 5, pp. 10-12.
(76) P. GASSENDI, *Opera Omnia*, Lyons 1658, Vol. 1.
(77) *Op. cit.* Section 1, Book II, pp. 192-216.
(78) *Op. cit.* Section 1, Book III, pp. 229-282.

and upon which the things in the universe are built, (79) and he lays down the conditions that must be satisfied by it. He then reviews the opinions of the ancient philosophers. Taking in turn first Homoiomerae, then the elements Water, Earth, Fire and Air, and next the Alchemical principles Salt, Mercury and Sulphur, he analyses these conceptions and thus shows what the *Prima Materia* is not. (80) He then deals with the Epicurean school (81) and reaches the conclusion that the *Prima Materia* must be represented by Atoms.

GASSENDI develops the fundamental properties of the atoms along Epicurean lines, holding that they cannot be created or destroyed ; (82) that they are solid (83) ; and that they cannot be divided into any smaller parts. (84) He makes it clear that this indivisibility does not mean that they are without size like a geometrical point, (85) but that they are real entities of extremely small dimensions and must not be conceived merely as mental concepts. His arguments are turned against the confusion of what he calls the *punctum physicum* and the *punctum mathematicum*.

In discussing the properties of atoms, (86) GASSENDI notes that DEMOCRITUS gave the atom only magnitude and figure, while EPICURUS added weight. (87) He would retain all three attributes. To give some idea of their size, he draws attention to the extremely small animals which, despite their minuteness, are equipped with their essential organs. These may be seen by means of a microscope, (88) and he points out that from such observation, some conception of the minuteness of the atom may be obtained. In connection with the form of the atoms, (89) his opinion is that it is very varied. Among other reasons for this belief he points out that no two grains of wheat, no two leaves of the same tree, etc., are identical. (90)

(79) *Op. cit.* Section 1, Book III. Chap 1, pp. 229-234.
(80) *Op. cit.* Section 1, Book III, Chaps. 2, 3 and 4, pp. 234-256.
(81) *Op. cit.* Section 1, Book III, Chap. 5, pp. 256-266.
(82) *Op. cit.* Section 1, Book III, Chap. 5, p. 259.
(83) *Op. cit.* Section 1, Book III, Chap. 5, pp. 260, 261.
(84) *Op. cit.* Section 1, Book III, Chap. 5, p. 261.
(85) *Op. cit.* Section 1, Book, III, Chap. 5, pp. 263 *et seq.*
(86) *Op. cit.* Section 1, Book III, Chaps. 6 and 7, pp. 266-279.
(87) *Op. cit.* Section 1, Book III, Chap. 6, p. 266.
(88) *Op. cit.* Section 1, Book III, Chap. 6, pp. 268-269.
(89) *Op. cit.* Section 1, Book III, Chap. 6, pp. 269-272.
(90) *Op. cit.* Section 1, Book III, Chap. 6, p. 270.

The last attribute of the Atom discussed by GASSENDI is its weight. (91) Unless the atom have this quality he considers that it would be impossible to give an explanation of things. (92) It is by reason of their weight that atoms are able to move among themselves, and he considers it as a property, or power, or innate permanent tendency which pushes the atoms into action. (93) This ability to move on the part of the atoms is considered of great importance, and is examined at length. (94)

GASSENDI, in opposition to EPICURUS, denies that atoms are eternal, unproduced, and moving of their own accord through infinite space. For him, the world is an organised whole ; God the creator and « first cause », above and beyond the physical world. (95) The self movement of the atoms is not « a se ipsis » but only « Dei gratia ».

§ 11. *Charleton.*

The system of GASSENDI is the basis of an English work (96) by WALTER CHARLETON (1619-1707), physician to King CHARLES II and one of the early Fellows of the Royal Society.

His *Physiologia* is divided into four books. Book I deals with the Universe, Time, Eternity, Place and a Vacuum : Book II deals with Atoms, their properties, size, shape and motion : Book III deals with the manner and reason of vision ; the nature of colours, light, sound, smells and taste ; the physical characteristics of substance, softness, hardness, ductility, fluidity, etc. ; an explanation of the behaviour of the loadstone and a most amusing chapter wherein « occult qualities are made manifest » : Book IV deals with Motion and Generation and Corruption. Each book is sub-divided into chapters, sections and articles, and the opening chapter in Book II is styled « The existence of Atoms, evicted ». (97) An early article (98) in the chapter claims that

(91) *Op. cit.* Section 1, Book, III Chap. 7, pp. 273-279.
(92) *Op. cit.* Section 1, Book III, Chap. 7, p. 273.
(93) *Op. cit.* Section 1, Book III, Chap. 7, p. 273.
(94) *Op. cit.* Section 1, Book V, p. 338, *et seq.*
(95) *Op. cit.* Section 1, Book IV, pp. 287-295.
(96) W. CHARLETON, *Physiologia Epicuro-Gassendo-Charltoniana or a fabrick of science natural upon the hypothesis of atoms*, London 1654.
(97) *Op. cit.* Book II, Chap. 1, p. 84.
(98) *Op. cit.* Book II, Art. 3, p. 85.

atoms have the attributes of the First Matter, and contains the following statement in support of the claim : « Nothing can be the Root or beginning of Material or Physical extension, but something so minute and solid, that nothing can be conceived more exiguous and impatible in nature ».

He tells us « that there are such things as Atoms or Insectile Bodies, in Rerum Natura cannot be long doubted by any judicious man, who shall thus reason with himself — Nature can produce Nothing out of Nothing, nor reduce anything to Nothing, is an Axiom, whose tranquility was never yet disturbed », (99) and « that Nature in her dissolution of concretions doth descend to insensible particles, that she cannot run on to Infinity, but must consist in Atoms, the term of exsolubility » (100).

He then seeks to show (101) that no physical continuum is infinitely divisible, and discusses fully the question of Atoms being « the first and universal matter ». (102) The fact that the number of elements had been a point of dissension among the ancients is brought forward, and it is claimed that while all the earlier ideas « are made equally plausible by a parity of specious arguments, it cannot appear either a defect of judgment, or an affectation of singularity in DEMOCRITUS and EPICURUS to have suspected them all of incertitude, and founded their physiology on an hypothesis of one single principle, Atoms ». (103) Then follows the statement of a number of reasons why « though the four vulgar elements may be the father, yet they cannot be the grandfather principle to all concretions » (104) and an explanation as to the difference between Atoms and Homoiomerae of ANAXAGORAS. (105) The section is concluded with a long dialogue between atomist and anti-atomist in which the difficulties of an atomic view-point are elucidated. (106)

The essential properties of the atoms are further dealt with. (107) « All atoms being equally corporeal and solid, must be

(99) *Op. cit.* Art. 7, p. 87.
(100) *Op. cit.* Arts. 8, 9 and 10, pp. 88 and 89.
(101) *Op. cit.* Chap. II, pp. 90-98.
(102) *Op. cit.* Chap. III, pp. 99-110.
(103) *Op. cit.* Book II, Chap. III, Art. 4, p. 101.
(104) *Op. cit.* Book, II Chap. III, Art. 6, p. 102.
(105) *Op. cit.* Book II, Chap. III, Art. 7, p. 102.
(106) *Op. cit.* Book II, Chap. III, Art. 8, pp. 103-106.
(107) *Op. cit.* Book II, Chap. IV, pp. 111-126.

substantially identical, or of one and the same nature, knowing no disparity of essence. » (108) The question of their « figure » is also discussed, (109) as « an essential adjunct of their quantity. For, insomuch as atoms are most minute bodies... they must have real dimensions and .... a determinate figure ». The atoms not only have a plain figure, but a solid one, whilst they also possess gravity or weight. With respect to the last three properties, it is admitted that there may be a difference between the atoms as to size, shape and weight. The question of the magnitude of the atoms comes under notice, (110) and it is made clear that the assumption of magnitude as the first essential property of atoms does not mean that they possess any sensible bulk, but that they are entities, realities endowed with certain corporeal dimensions, and not mathematical points. (111) A calculation as to the number of « elemental » atoms in a grain of frankincense is attempted, based on the result of its vaporisation, and the conclusion arrived at is that it contains at least 777,600,000, 000,000,000, « and so we may guess at the exiguity of a single atom ». (112) Futher conjecture as to the size of the atoms is made based upon the diffusion of a grain of vermillion dissolved in water : upon the small quantity of oil consumed by the flame of a lamp in a quarter of an hour, and upon the use of the microscope in discerning the minute particles of bodies, the smallest of which, according to ARCHIMEDES (*in arenario*) consists of ten hundred thousand millions of insensible particles. (113)

In treating of the Figures of Atoms (114) he tells us that « the atoms of the vulgar », seen in a sunbeam viewed through a microscope, are found to have « many irregular and dissimilar appearances » as have the real atoms. The variation of « Figures in Atoms » is substantiated by comparison with natural substances of the same species, in which case no two are found to be exactly alike. (115)

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(108) *Op. cit.* Book II, Chap. IV, Sect. 1, Art. 2, p. 111.
(109) *Op. cit.* Book II, Chap. IV, Sect. I, Art. 2, pp. 111 and 112.
(110) *Op. cit.* Book II, Chap. IV, Sect. II, pp. 113-116.
(111) *Op. cit.* Book II, Chap. IV, Sect. II, Art. I, p. 113.
(112) *Op. cit.* Book II, Chap. IV, Sect. II, Art. 4, p. 114.
(113) *Op. cit.* Book II, Chap. IV, Sect. II, Arts. 7, 8 and 9, p. 11 .
(114) *Op. cit.* Book II, Chap. IV, Sect. III, pp. 117-121.
(115) *Op. cit.* Book II, Chap. IV, Sect. III, Art. 4, p. 118.

The final section to this book (116) is concerned with the motions of atoms and the material Epicurean standpoint is replaced by the idea that the atoms were created by God, given an internal energy by him and that « their internal motive virtue necessitates their perpetual commotion among themselves ». (117)

The date of the publication of CHARLETON's work (1654) brings us to the age of the foundation of the Academies and the work of ROBERT BOYLE. The atomic theory now emerges into fuller light, but its discussion must be postponed. Here we have only sought to demonstrate a continuous atomic tradition from the first half of the fifteenth to the second half of the seventeeth century.

(London) G. B. STONES.

(116) *Op. cit.* Book II, Chap. IV, Sect. IV, pp. 121-126.
(117) *Op. cit.* Book II, Chap. IV, Sect. IV, p. 126.

The Relation of Plana and Bravais to Theory of Correlation.

The name of AUGUSTE BRAVAIS is thoroughly familiar to most students of statistics, although his writings are not generally accessible in America and have probably not been widely read. Most of us are aware that the celebrated paper which he read before the Institut de France in 1846 is often referred to as containing the first exposition of correlation theory, that in this memoir he gave a formula for the normal surface of probability for two variables, and that this formula contains the product term. Thus he set forth the mathematics of the normal correlation surface three decades before the idea of correlation had been conceived. It is not generally known, however, that a similar result had been reached thirty-four years earlier by an Italian astronomer GIOVANNI PLANA. No writer on correlation has linked the name of PLANA with that theory, and but few have mentioned his work on probability. In the very extensive bibliography at the close of « Die Entwicklung der Wahrscheinlichkeitstheorie und ihrer Anwendungen,» (1) CZUBER gives the title of one of his papers on probability, as does also KEYNES in the bibliography in his *Treatise on Probability* (2). MERRIMAN mentions two of them in his catalogue (3) of works on the subject of Least Squares.

This list contains the titles of four hundred and eight books and papers, three hundred and twelve of which MERRIMAN indicates that he has read. It so happens that the particular memoir of 1812 in which PLANA developed the equation for the probability

(1) *Jahresbericht der Deutscher Mathematiker-Vereinigung*, Vol. VIII, Leipzig, 1899.

(2) London, 1921.

(3) « A List of Writings Relating to the Method of Least Squares, with Historical and Critical Notes », *Transactions of the Connecticut Academy of Arts and Sciences*, Vol. IV, New Haven, 1877-1882, pp. 151-232.

of the simultaneous occurrence of given values of two variables is one of the remaining ninety-six which MERRIMAN had not been able to examine, while he had read the far less interesting paper of 1818. One would expect that the long and scholarly treatment of Wahrscheinlichkeitsrechnung by CZUBER in the *Encyklopädie der Mathematischen Wissenschaft* (4) would furnish information concerning both PLANA and BRAVAIS. Yet even here PLANA's name does not occur, and for BRAVAIS there is but a single sentence of four lines stating that he showed that points of equal probability lie on similar, concentric, and similarly placed ellipses or ellipsoids.

The purpose of this paper is to describe PLANA's general method of approach to the equation for the surface of normal probability, to present his formula, and to compare both with the later work of BRAVAIS. By way of background, we will first summarize briefly the contributions of GAUSS and ADRAIN to the same subject.

THE WORK OF ADRAIN. In 1808 ADRAIN published two proofs of the « Law of Error » in a brief article entitled « Research concerning the probabilities of the errors which happen in making observations, &c. » (5). The second of these proofs is less elegant than the first, but more interesting from the point of view of this discussion, because in it he considers the probability of the simultaneous occurrence of two independent errors in the position of a point, the directions of the errors being perpendicular to each other. The object of his search is the curve of equal probability. This he finds to be a circle when the two « sources of error » are equal, and to be an ellipse when the « equally probable errors » are in any given ratio other than one to one. The axes of the ellipse coincide with the coordinate axes. These curves are exactly the contour lines of our surface of normal probability for uncorrelated variables, the ellipse being elongated in the direction of the variable with the greater dispersion. ADRAIN has no product terms, he does not even express the probability along this curve as an ex-

(4) Leipzig, 1904.

(5) *The Analyst or Mathematical Museum*, Vol. I.

MERRIMAN reproduces a large part of the second proof.

See also COOLIDGE : « ROBERT ADRAIN, and the beginnings of American Mathematics », *American Mathematical Monthly*, Vol. XXXIII, February 1925, p. 61-75.

Through the courtesy of the Yale library I have been able to consult a copy of the original.

ponential, although this is undoubtedly in his mind, and he has no interest in the relation between the variables except to specify that they are to be considered independent. Contrary to the statement of Professor COOLIDGE (6), ADRAIN did not give the first proof of the law of error. That had been done by DE MOIVRE in 1733. ADRAIN does appear to deserve the credit for being the first to consider the probability that two errors will occur together, a fact which Professor COOLIDGE does not mention.

THE WORK OF GAUSS. The exponential law of error had been established by DE MOIVRE in 1733, but his work (7) was little known. In 1783, LAPLACE (8) gave the expression for the probability integral and suggested the desirability of its tabulation. Acknowledging his debt to LAPLACE (9), GAUSS made use of this same principle in Section III of Book II of *Theoria Motus*, and reached the principle of least squares, stating it thus, with italics as indicated : ***Thus the most probable system of values of the unknowns p, q, r, s, etc. will be that one in which the sum of the squares of the differences between the observed and computed***

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(6) « Here we have the first known demonstration of the exponential law of error, published a year later by GAUSS, and usually associated with his name, » p. 67 *op. cit.* « Nothing can take away from ADRAIN the credit of having been the first to face squarely the problem of deducing a general law for the distribution of errors, and of carrying it through to the point of finding a formula which, while not perfect, and not always applicable, is still the best that has been devised. » p. 69.

(7) ABRAHAM DE MOIVRE : *Approximatio ad Summam Terminorum Binomii* $(a + b)^n$ *in Seriem expansi.*

PEARSON : « Historical Note on the Origin of the Normal Curve of Errors, » *Biometrika* XVI, 1924, pp. 402-404.

ARCHIBALD : Communication to *Nature*, April 17, 1926.

PEARSON : Communication to *Nature*, April 17, 1926.

ARCHIBALD : « A rare Pamphlet of Moivre and some of his Discoveries, » *Isis*, Nov. 1926, VIII, 671-683.

(8) See PEARSON : « Notes on the History of Correlation », *Biometrika*, Vol. XIII, 1920-21, p. 26.

(9) « ... et quum per theorema elegans primo ab ill. LAPLACE inuentem, integrale $\int e^{-hh\Delta\Delta} d\Delta$ a $\Delta = -\infty$ usque ad $\Delta = +\infty$, fiat $\sqrt{\pi}/h$, (denotando per π semi-circumferentiam circuli cuius radius 1) functio nota fiet $\phi\Delta = \frac{h}{\sqrt{\pi}} e^{-hh\Delta\Delta}$.

Theoria Motus Corporum Cœlestium in Sectionibus Conicis Solem Ambientium, (Hamburg, 1809) Book II, Section III, § 177.

values of the functions V, V', V'', etc. is a minimum assuming that all the observations have the same degree of accuracy. (10). He says that he had used this principle since 1795, but that it was first published by LEGENDRE in *Nouvelles methodes pour la determination des orbites des comètes*, Paris, 1806. (11) Three years before the publication of *Theoria Motus*, GAUSS speaks of having used the method of least squares for twelve years, and says that he had not seen LEGENDRE's work, but he surmises from reports that they were using similar methods. (12)

It is in the *Theoria Combinationis Observationum Erroribus Minimis Obnoxiae*, 1823, and the *Supplementem* of 1826 that GAUSS developed the mathematics of the probability of the simultaneous occurrence of two or more errors ; hence these papers were antedated by ADRAIN's research of 1808 and PLANA's memoir of 1812. In his « Notes on the History of Correlation » cited above, Professor PEARSON has given an analysis of GAUSS's relation to the correlational calculus. As his paper is easily accessible it is unnecessary to go into detail here concerning GAUSS's treatment of the matter. Translating the unfamiliar symbolism of GAUSS into his own notation, PEARSON states that GAUSS found the probability of simultaneous occurrence of x_1, x_2,... to be proportional to $e^{-1/2\, u^2_{x_1,x_2,\ldots}}$ where $u^2_{x_1,x_2,\ldots}$ is equal to

$$S\left(\frac{A_1^2}{\sigma_a^2}\right)(x_1 - \bar{x}_1)^2 + S\left(\frac{B_1^2}{\sigma_a^2}\right)(x_2 - \bar{x}_2)^2 + 2\, S\left(\frac{A_1 B_1}{\sigma_a^2}\right)(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) + \ldots\ldots$$

«This is a normal surface which contains *the product terms*. As we now interpret it we say that the *x's* are *correlated* variates. And in this sense GAUSS in 1823 reached the normal surface of *n* correlated variates. But he does not seek to express all his relations in terms of

(10) « *Systema itaque maxime probabile valorum incognitarum p, q, r, s, etc. id erit, in quo quadrata differentiarum inter functionum V, V', V'', etc. valores obseruatos et computatos summam minimam efficiunt*, si quidem in omnibus obseruationibus idem praecisionis gradus praesumendos est. »

(11) The date usually given is 1806, but MERRIMAN states that he had a copy plainly marked 1805.

(12) «Il Comet vom Jahre 1805,» *Zach's Monatliche Correspondenz*, Vol. XIV, 1806, p. 181-186. See MERRIMAN, *op. cit.*

the S. D. 's σ_{x_1}, σ_{x_2}, σ_{x_3} ... and the correlations r_{12}, r_{13},... of these variates. These x-variates are not for GAUSS, nor for those who immediately followed him, the *directly* observed quantities... The point is this: that the Gaussian treatment leads (i) to a non-correlated surface for the directly observed variates, (ii) to a correlation surface for the *indirectly* observed variates. This occurrence of product terms arises from the geometrical relations between the two classes of variates, and not from an organic relation between the indirectly observed variates appearing on our direct measurement of them. It will be seen that GAUSS's treatment is almost the inverse of our modern conceptions of correlation. For him the *observed* variables are independent, for us the observed variables are associated or correlated. For him the non-observed variables are correlated owing to their known geometrical relations with observed variables ; for us the unobservable variables may be supposed to be uncorrelated causes, and to be connected by unknown functional relations with the correlated variables. In short there is no trace in Gauss's work of observed physical variables being - apart from equations of condition - associated organically, which is the fundamental conception of correlation. » (13)

THE WORK OF PLANA. GIOVANNI ANTONIO AMEDEO PLANA (1781 -1864) was born at Voghera, Italy. He was admitted to the École Polytechnique in Paris at the age of nineteen. At this time LAGRANGE and MONGE were teaching there, POISSON was just leaving the school, and DUPIN and ARAGO were just entering it. After three years he became a professor of mathematics in the École d'Artillerie, and like LEGENDRE served an apprenticeship studying the movement of projectiles before he came later to study the movement of heavenly bodies. Soon he formed a close friendship with the celebrated astronomer ORIANI, director of the observatory at Milan, and when a vacancy occurred, both ORIANI and LAGRANGE nominated him for the position of Professor of Astronomy at Turin. PLANA was given the chair in 1811, two years later was made director of the observatory, and soon thereafter was elected to the Academy of Sciences at Turin.

PLANA married Mlle. ALEXANDRA LAGRANGE, a niece of the

(13) Page 27, *op. cit.*

great JOSEPH LOUIS LAGRANGE, and thereafter she habitually signed her name Mme. PLANA-LAGRANGE.

VICTOR EMMANUEL I, pleased by the zest, the extensive knowledge, and the spontaneous wit of the young teacher, showed him many favors, and granted funds to transfer the observatory to a new building.

In 1811 an agreement was made to extend into Italy the French geodetic operations carried on by BIOT and ARAGO for establishing a meridian. When the preliminary calculations had been made, the mean of all the measures of a single angle was made the official value of that angle, and all the sides were recalculated. After that the triangulation of France was connected with the triangulation of Italy by a new triangulation. But the results of this vast geodetic operation did not accord with the astronomical observations with which they were compared, and the two astronomers on the commission, PLANA and CARLINI, went to Mt. Cenis to make new observations. They established an observatory in the grounds of an inn, set up a telescope, and an astronomical clock, and compared the time at Mt. Cenis with that at Milan by means of fire signals, producing a light by igniting a small amount of powder in a cannon. The Academy at Paris had followed this great piece of work with attention, and after the publication of the first volume, PLANA was made correspondent of the Academy in the section of geometry. After the second volume was issued, the Lalande prize was accorded to both PLANA and CARLINI. (14).

The *Catalogue of Scientific Papers of the Royal Society of London*, lists 133 memoirs by PLANA, beginning in 1809 and continuing until a few months before his death. Most of these deal with astronomy, although not a few relate to abstract problems in pure mathematics. Only three appear — from their titles — to have any direct bearing on the theory of probability. These are : (i) « Mé-

(14) These facts have been gathered chiefly from a paper entitled « Éloge Historique de JEAN PLANA, l'un des huit associés étrangers de L'Académie » by M. ÉLIE DE BEAUMONT, read in public session of the Academy Nov. 25, 1872. The addreesss made by M. FELIX CHIO and by Count FREDERICK SCLOPIS at a gathering held in his honor at the University of Turin were eulogistic rather than biographical. The nine letters now in the possession of Professor DAVID EUGENE SMITH, written by PLANA from Turin after the year 1828, contain no material related to probability, although they furnish some interesting personal details.

moire sur divers problèmes de probabilité » (15), (ii) « Soluzione generale di un problema di probabilità » (16) and (iii) « Allgemeine Formeln um nach der Methode der kleinsten Quadrate die Verbesserung von 6 Elementen zu bestimmen. » (17)

The « Allgemeine Formeln » is an elaboration of an idea previously set forth by GAUSS on the method of deriving certain equations according to the method of least squares. It contains no material related to the problem of the simultaneous occurrence of two or more variables. In passing, however, it is interesting to note the use of the term *wahrscheinlicher Fehler*, which occurs with no explanation of its meaning, as if the expression *probable error* were already well understood among German astronomers.

« Soluzione generale », received by the academy in July 1815 but not published until 1820, is also entirely free of any reference to the probability of the simultaneous occurrence of two variables, although it does make use of the normal law of error for a single variable. This paper contains some of the material previously published in 1813 in « Divers Problèmes », but not the part in which we are chiefly interested here. Evidently PLANA did not cherish highly his discovery of a formula which becomes extremely interesting in the light of modern correlation theory.

We will now present a brief summary of the material contained in the « Mémoire sur Divers Problèmes de Probabilité », to suggest the general method of approach.

PLANA begins by saying that when one hears for the first time the enunciation of the problem stated in this memoir, he probably thinks it more a matter of curiosity than use ; but on examining it closely, he soon recognizes that its chief use is to show the principles relative to the method of choosing a mean from the results of a set of observations. In that connection it ought to attract the attention of astronomers and physicists.

Imagine a polyhedron with $2n$ faces, numbered 1, 2, 3,...,n,

(15) *Mémoires de l'Académie Impériale des Sciences, Littérature et Beaux-Arts de Turin, pour les Années* 1811-1812, XX, Turin, 1813, pp. 355-408. This paper was read in the session of November 30, 1812.

(16) *Memorie di Matematica e di Fisica della Società Italiana delle Scienze, residente in Modena*, XVIII, 1820, pp. 31-45.

(17) *Zeitschrift für Astronomie und verwandte Wissenschaften*, VI, Tübingen, Nov. and Dec. 1818, pp. 249-264.

-1, -2, -3,.....$-n$. What is the probability of exposing a sum equal to zero on a single throw of P similar dice ?

The problem can be solved by expanding

$$(x^{-n} + x^{-(n-1)} + \ldots + x^{-2} + x^{-1} + x^{1} + x^{2} + \ldots + x^{n-1} + x^{n})^{P} = X^{P}$$

and taking the term independent of x. In the most simple case where $n = 1$ and $P = 2q$, the value of the coefficient sought is

$$\frac{(q+1)(q+2)(q+3) \ldots\ldots 2q}{1 \cdot 2 \cdot 3 \ldots\ldots q} = \frac{2^{2q}}{\sqrt{q\pi}}\left(1 - \frac{1}{8q} + \frac{1}{128q^2} - \text{etc.}\right).$$

Since the term in which we are interested is independent of x, it is permissible to set $x = e^{\omega\iota}$. The polynomial now becomes

$$X^{P} = (2\cos\omega + 2\cos 2\omega + 2\cos 3\omega + \ldots\ldots + 2\cos n\omega)^{P} \quad (1)$$

$$= 2^{P}(A + A'\cos\omega + A''\cos 2\omega + \ldots.) \quad (2)$$

Now $\int_{\omega=0}^{\pi} 2^{P}(A + A'\cos\omega + A''\cos 2\omega + \ldots.)\, d\omega = 2^{P} A\pi$

If Y is the coefficient independent of ω in equation (1), then

$$Y = \frac{2^{P}}{\pi} \cdot A\pi = \frac{2^{P}}{\pi}\int_{\omega=0}^{\omega} d\omega\,(\cos\omega + \cos 2\omega + \cos 3\omega + \ldots + \cos n\omega)^{P}.$$

The maximum value of $\cos\omega + \cos 2\omega + \cos 3\omega + \ldots + n\omega$ is n when $\omega = o$. We will let t be defined by the equation

$$(\cos\omega + \cos 2\omega + \cos 3\omega + \ldots + \cos n\omega)^{P} = n^{P} e^{-t^2} \quad (3)$$

Then $Y = \frac{(2n)^{P}}{\pi}\int d\omega \cdot e^{-t^2}$, and ω may be considered a function of t defined by equation (3). To find the limits of t, we note that when $\omega = 0$, $t = 0$, and that when $\omega = \pi$, $0 = n^{P} e^{-t^2}$ if n is even,

and $(-1)^{P} = n^{P} e^{-t^2}$ if n is odd.

In the latter case, if n is even, $t = \infty$, and if n is odd t approaches ∞ as a limit as P becomes indefinitely large. The limits of integration are therefore from $t = 0$ to $t = \infty$.

PLANA later makes the problem more general by asking what

is the probability that the sum of the numbers exposed on the dice shall equal q, and applies the results to a problem concerning the probable sum of the orbital inclinations of a set of planets. He says his results accord well with those given by LAPLACE in the *Mémoires de l'Institut*, 1809, p. 374. A similar derivation, somewhat simplified, may be found in English in *Theory of Errors of Observations and the Combination of Observations* (London, 2nd ed. 1875) by Sir GEORGE BIDDELL AIRY, Astronomer Royal of England.

The foregoing owes a good deal to LAPLACE. The part of the paper in which we are primarily interested at present begins on page 376 with a question as to what is the probability that the two equations

$$q'\epsilon' + q''\epsilon'' + q'''\epsilon''' + \ldots + q^{(p)}\epsilon^{(p)} = Q,$$
$$q_{,}\epsilon' + q_{,,}\epsilon'' + q_{,,,}\epsilon''' + \ldots + q_{(p)}\epsilon^{(p)} = Q'.$$

shall be satisfied simultaneously. This is only another way of asking what is the probability that in one throw of P dice the sum Q shall be exposed, while in another throw of the same number of similar dice, the sum exposed shall be Q'.

$$\text{Let } X' = x^{-nq_1}y^{-nq_,} + x^{-(n-1)q'}y^{-(n-1)q_,} + \ldots + x^{-2q'}y^{-2q_,} + x^{-q'}y^{-q_,} + x^{0'}y^{0_,} + x^{2q'}y^{2q_,} + \ldots + x^{(n-1)q'}y^{(n-1)q_,} + x^{nq'}y^{nq_,}$$

$$\text{and } X'' = x^{-nq''}y^{-nq_{,,}} + x^{-(n-1)q''}y^{-(n-1)q_{,,}} + \ldots + x^{-2q''}y^{-2q_{,,}} + x^{-q''}y^{-q_{,,}} + x^{0''}y^{0_{,,}} + x^{2q''}y^{2q_{,,}} + \ldots + x^{(n-1)q''}y^{(n-1)q_{,,}} + x^{nq''}y^{nq_{,,}},$$

etc.

If we develop the function $X' \cdot X'' \cdot X''' \ldots X^{(p)}$, the coefficients of $x^Q y^{Q'}$ in the expansion will give the desired probability. Again we will set $x = e^{\omega\iota}$ and $y = e^{\omega'\iota}$.

Then $X' = 2\Sigma \cos n\,(q'\omega + q_{,}\omega')$,

$X'' = 2\Sigma \cos n\,(q''\omega + q_{,,}\omega'')$, etc.

The coefficient of $x^Q y^{Q'}$ is the term independent of ω and ω' in the product $\cos(Q\omega + Q'\omega')\, X' \cdot X'' \cdot X''' \ldots X^{(p)}$.
Let z = this coefficient. With the exception of z, any term in the product is of the form $A \cos(\alpha\omega + \beta\omega')$,

or $$B \cos M\omega, \text{ or } C \cos N\omega'.$$

But $$\int_{\omega=-\pi}^{\pi} A \cos(\alpha\omega + \beta\omega')\, d\omega = 0.$$

$$\int_{\omega=-\pi}^{\pi} B \cos M\omega \, d\omega = 0,$$

and $$\int_{\omega'=-\pi}^{\pi} C \cos N\omega' \, d\omega' = 0.$$

Then $$z = \frac{1}{4\pi^2}\int_{-\pi}^{\pi} d\omega' \int_{-\pi}^{\pi} d\omega \cos(Q\omega + Q'\omega') X' X'' X''' \ldots X^{(p)}$$

since in that double integration all the terms disappear except z. By many transformations he shows that

$$z = \frac{(2n)^P}{4\pi^2}\int d\omega' \int d\omega \cos(Q\omega + Q'\omega')\, e^{-(a A\omega^2 + 2a B\omega\omega' + a C\omega'^2)}$$

and finally that

$$z = \frac{1}{4\pi a\sqrt{E}}\, e^{-\frac{1}{4aE}(CQ^2 - 2BQQ' + AQ'^2)}$$

We are immediately and forcibly struck by the similarity of this formula to the formula for ordinates of the normal correlation surface. Writing the latter as

$$y = \frac{1}{2\sigma_1\sigma_2\pi\sqrt{1-r^2}}\, e^{-\frac{1}{2(1-r^2)}\left\{\frac{x_1^2}{\sigma_1^2} - \frac{2r x_1 x_2}{\sigma_1\sigma_2} + \frac{x_2^2}{\sigma_2^2}\right\}}$$

we see that

$$E = 1 - r^2,\ C = \frac{\sigma_2}{\sigma_1}\ A = \frac{\sigma_1}{\sigma_2}\ a = \frac{\sigma_1\sigma_2}{2} \text{ and } B = r.$$

PLANA attaches no special significance to the letter B. Later (page 381) he considers the probability that three equations shall be satisfied simultaneously, and finds

$$z = \frac{(2n)^P\, e^{-N}}{8\pi\sqrt{\pi}\sqrt{GH}}$$

where N is a long polynomial containing the product terms.

He closes the paper with the remark that since the function

$\frac{h \cdot dx}{\sqrt{\pi}} e^{-h^2x^2}$ occurs in the solution of each of the problems discussed in the memoir, he is forced to suppose that the probability of errors of observation is represented by that formula.

PLANA is the second mathematician to treat of the probability that two errors should occur together, the first to discuss the probability of the occurrence of three, and the first to discuss the probability of the occurrence of dependent variables. «Divers Problèmes » is the first paper to show the product terms in the expression for that probability. His is the only use of a single symbol equivalent to our coefficient of correlation in the equation for the surface of normal probability before the time of the now famous incident (18) when, at Sir FRANCIS GALTON's request, Mr. J. D. HAMILTON DICKSON wrote the equation for the correlation surface.

THE WORK OF BRAVAIS. AUGUSTE BRAVAIS (1811-1836) was successively a lieutenant in the French navy, Professor of Astronomy in the Faculty of Science at Lyons, and Professor of Physics in the École Polytechnique in Paris. In 1854 he was made a member of the Institut de France. In the interests of scientific research he made numerous journeys and published his observations in a long succession of memoirs dealing with astronomy, with geodesy, with optics, and with physics. The *Catalogue of Scientific Papers of the Royal Society of London* gives a list of seventy six such memoirs, ranging in date from 1837 to 1856. Of all these, only one article — and that the one we are about to discuss — relates to probability. Shorter lists of his writings, such as those given by POGGENDORFF and LAROUSSE, omit this paper entirely, and it seems probable that but for GALTON's epoch making discovery of correlation in the seventies, BRAVAIS's one contribution to the theory of probability might have remained forever in the dim limbo of things forgotten.

BRAVAIS's celebrated paper « Analyse mathématique sur les probabilités des erreurs de situation d'un point » appeared in 1846 in volume 9 of *Mémoires de l'Institut de France*, pp. 255-332. The tenor of the discussion is geometric, and is thus in direct

(18) See GALTON : *Memories of My Life*, pp. 302-303, or PEARSON, *op. cit.* p. 37.

contrast to the treatment presented by PLANA, who suggests no spatial conformations at all. BRAVAIS begins by saying that the determination of the position of a point in space depends upon at least three elements ; when the point is restricted to a given plane the number of elements is at least two. The observation of these elements consists essentially in the measuring of lines or angles, or possibly time. Unavoidable errors in the different measures have an effect upon the position of the point. He has investigated the influence which elementary errors may exert on these possible errors of position, always confining himself to the case in which the elements are determined by a very large number of observations. If we call the observed elements a, b, c, ... and the coordinates of the point x, y, z,... we have in general

$$\left.\begin{aligned} x &= \phi\ (a,\ b,\ c,\ \ldots) \\ y &= \psi\ (a,\ b,\ c,\ \ldots) \\ z &= \chi\ (a,\ b,\ c,\ \ldots) \end{aligned}\right\} \quad (1)$$

It is plainly stated here that it is the *observed* elements a, b, c,... which BRAVAIS considers independent, while the related variables $x, y, z,\ldots$ are not obtained by direct measurement but by an analytic relation to the observed elements. This is exactly the reverse of our conception of correlation, as Professor PEARSON has pointed out. (19). In correlation the variables upon which we make direct observations are those whose relationship we wish to study, and we assume (in normal correlation at least) that each of them is a function of a number, perhaps a very large number, of independent variables not themselves susceptible of direct measurement.

BRAVAIS then goes on to say that if we differentiate the equations (1), taking a, b, c, ... as the errors made in each element, and x, y, z, as the resulting errors in the coördinates, we will have, if we neglect the terms of the second degree,

(19) PEARSON, *op. cit.* The outline given here was made directly from BRAVAIS's memoir. It is necessarily similar to that given by Professor PEARSON in « Notes on the History of Correlation, » except for the fact that it preserves the original notation whereas PEARSON sometimes departs from it, and that it reproduces some of BRAVAIS's explanatory comment which PEARSON does not quote. To afford a comparison with PLANA's treatment, it has seemed necessary to present such a summary here, even though that involves a repetition of some items in the article just mentioned.

$$\left.\begin{aligned} x &= \frac{d\phi}{da}\delta a + \frac{d\phi}{db}\delta b + \frac{d\phi}{dc}\delta c \ldots\ldots = A\delta a + B\delta b + C\delta c \ldots\ldots \\ y &= \frac{d\psi}{da}\delta a + \frac{d\psi}{db}\delta b + \frac{d\psi}{dc}\delta c \ldots = A'\delta a + B'\delta b + C'\delta c \ldots \\ z &= \frac{d\chi}{da}\delta a + \frac{d\chi}{db}\delta b + \frac{d\chi}{dc}\delta c \ldots = A''\delta a + B''\delta b + C''\delta c \ldots \end{aligned}\right\} (2)$$

He says that LAPLACE had derived the expression for the probability of an error when the number of elements is very large, and had treated exhaustively the case of errors which are linear functions of the elementary errors. BRAVAIS proposes to supplement the work of LAPLACE by considering the equations for the squares of errors such that

$$(\delta s)^2 = (\delta x)^2 + (\delta y)^2 + (\delta z)^2,$$

where δs is the total error in the position of a point, and by giving a geometric interpretation of the results.

BRAVAIS considers in turn the case where the point is assumed to lie on a given straight line, where it is restricted to a given plane, and where it is restricted to three-dimensional space. In the first case he points out that LAPLACE has shown, when $x = Am + Bn + Cp + \ldots\ldots$, that the probability that the dependent variable x should lie between x and $x + \delta x$ is of the form

$$\sqrt{\frac{h_x}{\pi}}\, e^{-h_x x^2}\, dx$$

where h_x is given by the equation

$$\frac{1}{h_x} = \frac{A^2}{h_m} + \frac{B^2}{h_n} + \frac{C^2}{h_p} \ldots\ldots$$

or, as we would write it now,

$$2\sigma_x^2 = 2A^2\sigma_m^2 + 2B^2\sigma_n^2 + 2C^2\sigma_p^2 + \ldots$$

In the case where the point is assumed to lie upon a given plane, he takes that plane for the plane of the coordinates x, y. The point as determined by the observations serves as origin ; x, y, are the coordinates of the « true point » as it would be determined by the observations freed from all error. Now we have

$$x = Am + Bn + Cp...$$
$$y = A'm + B'n + C'p...$$

Obviously these equations are identical with (2) when x is substituted for δx and m for δa. Also h_x is analogous to h_y and is given by the equation

$$\frac{1}{h_y} = \frac{A'^2}{h_m} + \frac{B'^2}{h_n} + \frac{C'^2}{h_p} \ldots\ldots$$

$$\text{Let } \alpha_0 = \frac{\Sigma A^2}{h_m} \text{ and } \alpha_1 = \frac{\Sigma A'^2}{h_m}$$

$$\text{Then } \frac{1}{h_x} = \frac{\Sigma A^2}{h_m} = \alpha_0 \text{ and } \frac{1}{h_x} = \frac{\Sigma A'^2}{h_m} = \alpha_1$$

The coexistence of the same variables $m, n, p,\ldots$ in the simultaneous equations in x and y produces a correlation such that the moduli h_x, h_y cease to represent the probability of the occurrence of the pair of values (x, y) in the true sense of the question. (20).

He eliminates the independent variables by creating a set of arbitrary dependent variables of the form

$$V = A^{(i)}m + B^{(i)}n + C^{(i)}p + \ldots$$

so that there are as many equations as independent variables. Then he finds that

$$dm\, dn\, dp = \frac{dx.\, dy.\, dz.}{A\, B'\, C' \ldots}$$

and that the differential of the probability ought to be

$$\frac{1}{A\, B'\, C'' \ldots} \sqrt{\frac{h_m h_n h_p}{\pi\, \pi\, \pi}}\, e^{-\left\{h_m \Sigma^2 ax + h_n \Sigma^2 a'x + \ldots\right\}}\, dx\, .\, dy \ldots dw\, .$$

After some further transformations he reaches the equation

(20) « La coexistence des mêmes variables $m, n, p\ldots$ dans les équations simultanées en x et y, amène une corrélation telle, que les modules hx, hy, cessent de représenter la possibilité des valeurs simultanées de (x, y) sous le vrai point de vue de la question. p. 263.

$$\frac{d\omega^2}{dx \cdot dy} = \frac{K}{\pi} e^{-(ax^2+2cxy+by^2)},$$

where $\frac{d\omega^2}{dx\,dy}$ is the probable frequency over the element $dx\,dy$.

Integrating from $x = -\infty$ to $x = +\infty$, and in like fashion integrating for y, he finds

$$a = K^2\alpha_1,$$
$$b = K^2\alpha_2,$$

when we compare these results with the form in which we usually write the equation now :

$$y = \frac{1}{2\sigma_1\,\sigma_2\,\pi\sqrt{1-r^2}} e^{-\frac{1}{2(1-r^2)}\left\{\frac{x_1^2}{\sigma_1^2} - \frac{2r\,x_1x_2}{\sigma_1\sigma_2} + \frac{x_2^2}{\sigma_2^2}\right\}},$$

we see that

$$K = -\frac{1}{2\sigma_1\,\sigma_2\sqrt{1-r^2}}.$$

$$a = \frac{1}{2\sigma_1^2(1-r^2)}$$

$$b = \frac{1}{2\sigma_2^2(1-r^2)}$$

$$e = -\frac{r}{2\sigma_1\,\sigma_2(1-r^2)},$$

$$K^2 = \frac{1}{4\sigma_1^2\sigma_2^2(1-r^2)^2}.$$

Obviously BRAVAIS had no single term equivalent to our coefficient of correlation.

BRAVAIS calls attention to the fact that if the exponent

$$(ax^2 + 2\,e\,x\,y + b\,y^2)$$

is a constant, the equation

$$ax^2 + 2\,e\,x\,y + b\,y^2 = \mathrm{D}$$

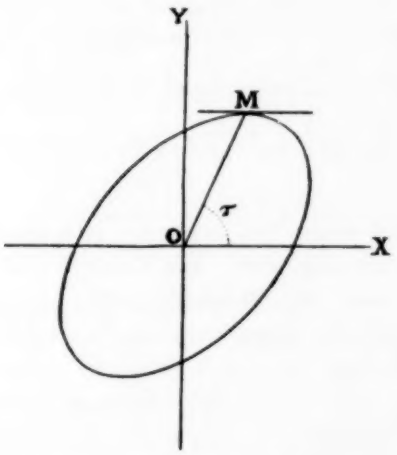
represents an ellipse with center at the origin, and with principal

axes which are in general different from the coordinate axes. By varying the constant we have an unlimited series of similar ellipses. To obtain the surface, we require the maximum value of y. If we differentiate $a\,x^2 + 2\,e\,x\,y + b\,y^2 = D$ with respect to y and set the result equal to zero, we will have

$$x = -\frac{e}{a}\,y \quad \text{and} \quad y^2 = \frac{aD}{ab - e^2}$$

The radius vector corresponding to these values of x and y will be the semi-conjugate diameter of the axis of x, and if τ represents the angle between the two diameters, the length of OM will be

$$\frac{1}{Sin\,\tau}\sqrt{\frac{aD}{ab - e^2}}$$



It will be apparent to the reader, though it was not so to BRAVAIS, that OM is also the line of regression of x on y and that the equation $x = -\frac{a}{e}\,y$ is the regression equation. BRAVAIS was unfortunately more concerned with the areas of the ellipses than with any relationship between the variables, and so remained blind to the stupendous idea in whose vicinity his mind was hovering. That he might, with one leap of creative imagination, have pounced squarely upon this conception of a measure of the relationship between x and y, is apparent from the discussion on page 279. It is equally apparent that he did not leap. On this page he suggests

that when the coordinates are transformed so that the axes of the ellipse coincide with the coordinate axes, the equation reduces to the form

$$\frac{d\omega^2}{dx\,dy} = \frac{1}{\pi}\sqrt{h_x h_y}\; e^{-(h_x x^2 + h_y y^2)}$$

upon which fact BRAVAIS makes the pertinent comment, « Ainsi, dans ce cas, la probabilité des valeurs simultanées $x = x, y = y$, est exactment la même que si les variables x et y étaient entièrement indépendantes l'une de l'autre. Ce cas se réalise, entre autres, si les équations... sont de la forme

$$x = Am + Bn + Cp + \ldots$$
$$y = A'm_1 + B'n_1 + C'p_1 + \ldots$$

les variables m, n, p,... de l'équation en x étant essentiellement distinctes des variables m, n, p... de l'équation en y. »

In the case of three variables, he gives the analogous expression

$$\frac{G}{\pi^{\frac{3}{2}}}\, e^{-(ax^2+by^2+cz^2+2exy+2fxz+2gyz)}\; dx\,dy\,dz,$$

and observes that when this exponent is a constant we have a series of concentric ellipsoids ; and further, that when these are so placed that their axes coincide with the coordinate axes, $e = f = g = 0$ and the expectation of frequency reduces to

$$\frac{G}{\pi^{\frac{3}{2}}}\, e^{-(ax^2+by^2+cz^2)}\; dx\,dy\,dz.$$

In his resume, BRAVAIS shows beyond the possibility of misunderstanding that he has no thought of applying his theory to any field except that specified in the title — errors in the geometric position of a point. Certainly he was not thinking of the strength of the relation between the variables, nor did he ever single this out for attention. There is nothing in the memoir to indicate that he had seen PLANA's « Divers Problèmes, » although it is reasonable to suppose that he must have been acquainted with his astronomical researches. Nor is there any mention of GAUSS or ADRAIN. We would scarcely expect him to have seen the single volume which was the beginning and the end of *The*

Analyst, but it is difficult to believe that any scholar who taught astronomy and made geodetic studies in Europe in the forties could have remained unacquainted with *Theoria Motus* and *Theoria Combinationis*.

SUMMARY

A comparison of the two memoirs brings out the following facts :

PLANA's paper appeared some thirty four years before that of BRAVAIS, and ten years before the *Theoria Combinationis* of GAUSS.

PLANA's paper seems to have been unknown to BRAVAIS and to most other writers on probability ; and BRAVAIS's paper appears to have received little attention until after GALTON published his *Natural Inheritance* in 1889.

PLANA treats the subject analytically, employing no geometric concepts at all. BRAVAIS is chiefly interested in the geometric properties of the frequency surface.

Each is considering the probability that a pair of equations of the form

$$X = ax + bx' + cx'' + \ldots$$
$$Y = ay + by' + cy'' + \ldots$$

should be simultaneously satisfied. For BRAVAIS, the *independent* variables, x, x', x''.... y, y', y''.... are the ones *directly observed*, and the *dependent* variables X and Y are *not directly observed*. In correlation theory we seek to measure the degree of relationship between *dependent* variables which we *observe directly*. PLANA makes no specific statement as to which variables are independent, or as to which are directly observed. In the particular situation which suggests this problem to him, it would be possible to consider that either the numbers on the separate faces of the dice or the sums Q and Q′ were the directly observed quantities.

Both PLANA and BRAVAIS generalize the theory to include the case of three dependent variables.

PLANA's formula has in it a single lettre, B, which exactly takes the place of r in the formula for the surface of normal correlation. BRAVAIS has no such single letter.

BRAVAIS gives a treatment which produces the mathematical

correlative of a regression line, a regression equation, and the ellipses of equal frequency of the probability surface. PLANA has nothing of the sort.

Neither writer conceived of this as a matter which could have applications outside the fields of astronomy, physics, geodesy.

Neither one saw that the strength of the relationship between the two dependent variables might be a matter of major importance, although BRAVAIS did recognize the fact that when the variables were unrelated the form of the equation would be altered. Neither one isolated the product term in the exponent for particular attention.

Neither author appears to have been sufficiently impressed with the importance of the theory to have returned to the subject. Even when again writing on probability three years later, PLANA does not repeat the equation in which we are chiefly interested, although he does review some portions of the first part of the memoir.

Either writer might have discovered correlation, but neither did.

New York HELEN M. WALKER.

N° 19. Fig. 25, 26. — LEVERRIER

N° 20. Fig 27, 28. — GULDBERG

Nº 21. Fig. 29, 30. — ALEX. AGASSIZ

N° 22. Fig. 31. — STOKES

N° 23. Fig. 32. — DUPIN

Medallic Illustrations of the History of Science.

(First Series : XIXth and XXth centuries)
Fourth article.

The third instalment appeared in *Isis*, vol. IX, p. 420-23, pl. 19-22, 1927. The Editor will welcome photographs or suggestions for the following parts of this series. It is intended to publish only medals having a sufficient scientific and artistic interest. Medals dedicated to scientists who are still living can not be published at once but will be considered for ulterior publication.

GEORGE SARTON.

No. 19. URBAIN JEAN JOSEPH LE VERRIER (or LEVERRIER). Born at Saint-Lô, Manche, 1811 ; died in Paris 1877. French astronomer and mathematician. Mathematical discovery of the planet Uranus in 1846.

Biography by JOSEPH BERTRAND read in the Académie des Sciences in 1879.

See also *Isis* I, 309.

Artist : ALPHÉE DUBOIS. French sculptor and medallist. Born Paris 1831 ; died Clamart, Seine, 1905. Review of his work by MAZEROLLE in *Gazette numismatique française*, 1906, 1-27.

Size : 68 mm. *Collection* : D. E. SMITH ; G. S.

Publisher : Monnaie, Paris.

Obv. : U. J. J. LEVERRIER de l'Académie des Sciences. ALPHÉE DUBOIS, 1884 (fig. 25).

Rev.: Allegorical figures of the sun, surrounded by the planets (fig. 26).

No. 20. CATO MAXIMILIAN GULDBERG. Born in Christiania, 1836 ; died in 1902. Norwegian mathematician and chemist. He is

remembered chiefly because of the work done in collaboration with PETER WAAGE : the mathematical study of the speed of chemical reactions and of equilibrium conditions (Études sur les affinités chimiques. Christiania, 1867).

See in *Ostwald's Klassiker*, no. 104 : Untersuchungen über die chemischen Affinitäten aus dem Jahren 1864, 1867, 1879. Übersetzt und hrg. von R. ABEGG. 182 p., 18 pl. Leipzig, 1899.

Artist : IVAR THRONDSEN, Norwegian medallist, born in 1853; residing in Kongsberg, Norway.

Size : 60 mm. *Collection* : The Numismatic Cabinet of the University of Oslo. Communicated by Dr. HANS OLST, director.

Obv. : CATO MAXIMILIAN GULDBERG. Født 11 Aug. 1836, død 14 Jan. 1902. (Portrait, fig. 27).

Rev. : For videnskabeligt arbeide (fig. 28).

No. 21. ALEXANDER (EMMANUEL) AGASSIZ. Son of LOUIS AGASSIZ. (Born in Neuchâtel, Switzerland, 1835 ; died at sea in 1910). American zoologist and oceanographer.

Biographical memoirs by GEORGE LINCOLN GOODALE (17 p. National Academy, Washington, portrait, 1912); — by ALFRED GOLDSBOROUGH MAYER (*Annual Report of Smithsonian Institution* for 1910, 447-72, portrait) — by Sir JOHN MURRAY (*Bull. of Museum of comparative zoology*, vol. 54, 137-58, Cambridge 1911); — by HENRI PICKERING WALCOTT (*Proc. American Acad.*, vol. 48, 31-44, Boston 1912). — G. R. AGASSIZ. Letters and recollections of A. A. xi+454 p., illustrated, Boston, 1913.

Artist : THEODORE SPICER- SIMSON. Anglo-French medallist. Born at Le Havre, France in 1871. One of the leading medallists of our time.

Size : 78 mm. *Collection* : American Numismatic Society, Communicated by Mr. HOWLAND WOOD ; New York.

Obv. : National Academy of Sciences (Portrait, fig. 29).

Rev. : ALEXANDER AGASSIZ Medal. Oceanography. (fig. 30).

No. 22. Sir GEORGE GABRIEL STOKES, Bart. (Born at Skreen, Co. Sligo, Ireland, 1819 ; died at Cambridge, 1903). British mathematician and physicist.

Mathematical and physical papers. 5 vols. Cambridge

1880-1905. Vol. 4 and 5 edited by J. LARMOR ; vol. 5 contains obituary by LORD RAYLEIGH.

Sir JOSEPH LARMOR. Memoir and scientific correspondence of STOKES. 2 vols. Cambridge 1907. — Obituaries by KARL VON VOIT (*Sitzungsber. d. bayer. Akad., math. Cl.*, vol. 33, 550-556, 1903) and by ERNEST WILLIAM BROWN (*Annual report of Smithsonian Institution*, 1904, 773-777).

Artist : GEORGE WILLIAM DE SAULLES. English medallist, sometime chief engraver of the Royal Mint. Born in 1862, died in 1903.

Date : 1899. *Size* : 64 mm.

Collection : American Numismatic Society, New York. Communicated by Mr. HOWLAND WOOD, New York.

Obv. : GEORGE GABRIEL STOKES. b. 1819. (Portrait, fig. 31).

Rev. : Viro illustri et de philosophia egregie merito GEORGIO GABRIELI STOKES, Newtoni cathedram apud Cantabrigienses annum jam L^M^ obtinenti et sua academia et multae academiae salutem amicissime precantur tam sibi quam ipsi de tali vita tanto ingenio gratulantes kal. Jun. A. S. MDCCCXCIX (Laurel branches).

No. 23. Baron (PIERRE) CHARLES (FRANÇOIS) DUPIN. Born in Varzy, Nièvre, France in 1784; died in Paris in 1873. French mathematician and naval engineer. He is chiefly remembered because of his studies on the geometry of surfaces (notion of conjugate tangents of a point of a surface and of indicatrix; Dupin's cyclide ; Dupin's theorem). Développements de géométrie, avec des applications à la stabilité des vaisseaux, etc. Paris, 1813.

Eloge historique par JOSEPH BERTRAND. *Mémoires de l'Académie des Sciences*, t. 44, 1888, p. i-xxvii (read in 1883).

Artist : (JEAN FRANÇOIS) ANTOINE BOVY. Swiss medallist. Born at Geneva in 1795, died there in 1877. (That is, according to L. FORRER. According to Nouveau Larousse Illustré, he was born in Geneva 1803, died in Paris 1877). Five other members of the BOVY family (three generations) were medallists and coin engravers and all of them worked for the Geneva mint. ANTOINE was the eldest of the second generation and the most prolific ; his activity was prodigious.

Size : 50 mm. *Collection* : D. E. SMITH, New York.

Publisher : Monnaie, Paris.

Obv. : Baron CHARLES DUPIN de l'Académie des Sciences. (fig. 32).

Rev. : A CHARLES DUPIN, après 50 années d'heureuse confraternité, les membres de l'Académie des Sciences, ses collaborateurs, ses amis, 1818-1868.

Another medal, having the same obverse, bears the following legend upon its back : « Né à Varzy 1784. De l'Acad^e des Sc^es 1818. Député du Tarn 1827. Sénateur 1852. (Also in D. E. SMITH's collection).

Notes and Correspondence

Queries and Answers.

Edited by TENNEY L. DAVIS (Mass. Institute of Technology, Cambridge, Mass.)

Question No. 9. — PAUL JOSEPH BARTHEZ ((1734-1806).

Existe-il, en dehors de la France, des manuscrits, des monographies ou des articles concernant PAUL JOSEPH BARTHEZ qui fut professeur et chancelier de la faculté de médecine de Montpellier ? En dehors des articles d'encyclopédie et des grands traités d'histoire de la médecine, je ne connais rien.

Dr. R. MOURGUE, Nîmes, France.

Question No. 10. — ***Tacuinum***. Can some one throw more light upon the medieval use of the word ***Tacuinum*** than I am about to offer? I have seen somewhere the Arabic original of the word (***takwim*** ?) translated in Latin as ***dispositio per tabellas***? The medical treatise of IBN BUTLÁN, who died about 1063, was arranged in 15 parallel columns and called *Taqwim al-sihha* (Tables of Health) — see G. SARTON, ***Introduction to the History of Science***, I (1927) 730-731. In Latin translations of the thirteenth century (see my ***History of Magic and Experimental Science***, II (1923) 757, note 2) the work had great influence, and it is my impression that this led sometimes to a careless use of the word ***Tacuinum*** for other kinds of medical treatises. But by the fifteenth century, if not before, we find the word also used for an astrological prediction. Sometimes this has some connection with medicine, as in the following manuscript. University of Bologna Library, MS. 2, 15th century, autograph, IOHANNIS PAULI DE FUNDIS Tacuinus astronomico-medicus, Incipit, « Altissimi Dei nostri Ihesus Christi virtute chooperante primo in hoc meo judiciolo, » Explicit, « Bonon. die septima febr. 1435 per doctorem artium IOHANNEM PAULUM DE FUNDIS actu legentem in astronomia et in medicina... et necnon inclite et excelse communitatis Bononie astrologum benemeritum. » But the work itself is a prediction of the events of the coming year, 1435, by this professor of astronomy and medicine and municipal astrologer of Bologna. Indeed, in 1495-1496 and 1496-1497 it was stated as one

of the duties of one of the three professors of astronomy in the University of Bologna that he should give astrological judgments « and *tacuinum* » (Et fiat iudicium et tacuinum) i.e. presumably, prepare a prediction for the ensuing year. (1) Of such predictions by Italian professors I have seen many specimens in the manuscripts. And in a Vatican manuscript we are plainly told that composing an almanac and a *Tacuinum* are the same thing : Vatic. lat. 3099, fol. 9, de scientia faciendi Almanach idest Tacuinum. Finally an item may be noted in a late manuscript of the sixteenth century at Munich : Cod. lat. Monac. 192, 1544 A. D., fol. 149, Ratio lunae secundum Tacuinum Iudeorum. But here the word has perhaps reverted towards its original significance and means astronomical tables.

LYNN THORNDIKE.

Answer No. 10. — *Tacuinum, taqwîm.* With a digression on the word almanac.

The translation of the word taqwîm (2) by the phrase ***dispositio per tabellas*** is very good. This word expresses the action of precising something and establishing it in the right manner; hence it was used to designate tables wherein our knowledge of a certain subject is exhibited clearly and synoptically. The idea of summing up one's knowledge in tabular form was very probably inspired by the astronomical and trigonometrical tables for which this form was a matter of necessity. However the word taqwîm was never used to designate astronomical tables; at any rate I do not know any such tables so called. Astronomical tables were named zîj (pl., aziyâj). But a passage of IBN KHALDÛN suggests how the word taqwîm may have come to mean table (Prolegomena, ed. QUATREMÈRE, vol. 3, 107; French translation, vol. 3, 148.) He says that the astronomical tables are called aziyâj, but that the determination of the position of the stars for a given time, i.e., the determination of the numbers occurring in these tables, is called ta'dîl and taqwîm.

The use of taqwîm to mean *dispositio per tabellas* can be best illustrated by quoting the titles of a few Arabic works. (I quote them in chronological order).

1) It is interesting to note that the earliest work entitled Taqwîm, of which I came across, was not a medical work as I expected, but a book on jurisprudence (fiqh), the Taqwîm al-adillat fî uṣûl al-fiqh, by ABÛ

(1) DALLARI, UMBERTO. *I Rotuli dei Lettori legisti e artisti dello studio bolognese dal* 1384 al 1799, 1888-1924, 4 vols. I cannot give the exact page reference now, but it can readily be found from the date.

(2) Arabic words are transliterated according to the system explained in *Isis* (6, 410-12) or in my *Introduction* (I, 46, 1927).

ZAID 'ABDALLÂH AL-DABÛSÎ, who is considered the founder of the 'ilm al-khilâf, which might be translated the science of controversial subjects, or more boldly, comparative law. This AL-DABÛSÎ died at Bukhârâ in 1038-39 (I ought to have included him in my *Introduction*, but failed to do so).

2) The Tables of health, Taqwîm al-ṣiḥḥa, of IBN BUṬLÂN, who died c. 1063 (*Introduction*, I, 730).

3) The Tables of bodies, Taqwîm al-abdân, of IBN JAZLA, who died in 1100 (*Introduction*, I, 772).

IBN JAZLA elaborated IBN BUṬLÂN's work; both were Christians of Bagdad. Their works are the only important medical works called taqwîm, but they were so popular East and West, that the word taqwîm or tacuinum took easily a medical connotation.

4) The Tables of understanding, Taqwîm al-dhihn, a logical synopsis, by ABÛ-L-ṢALT UMAIYA AL-ANDALUSÎ, who died at Tunis in 1134.

5) Another treatise on comparative jurisprudence, the Taqwîm al-naẓar, composed in 1167-68 by IBN AL DAHHÂN.

6) A geographical treatise, the well known Taqwîm al-buldân, completed in 1321 by ABÛ-L-FIDÂ'.

Finally, I shall still quote a work of a much later time — the seventeenth century — to show the persistence of this tradition.

7) A historical synopsis, the Taqwîm al-tawârîkh, written in Turkish by the famous bibliographer ḤÂJJÎ KHALFA, who died in 1658. (Italian translation printed at Venice, 1697).

A number of other books called Taqwîm will be found in ḤÂJJÎ KHALFA's bibliography (FLUEGEL's edition, vol. 2, 1837, nos. 3487-3504), but those already mentioned are more than sufficient to show that the word taqwîm refers to the form of the treatise, the tabular or synoptic form, rather than to the contents, for the tables which have been quoted deal with a variety of subjects : law, medicine, geography, and history. There are also grammatical works called taqwîm — e.g. the Taqwîm al-lugha by IBN AL-JAUZÎ (d. 1201) — but in this case taqwîm means more probably correction, rectification.

***

In modern usage taqwîm (pl., taqâwîm) means almanac; and this word is commonly used not only in Arabic, but also in Persian, Hindustani and Turkish. This suggests a new problem far more difficult than the first, — what may be the origin of the word almanac ? Of the three earliest Arabic glossaries quoted in my *Introduction* (vol. I, 782), the first, the Glossarium latino-arabicum (second half of the eleventh century ?) does not contain any such word, but the two others, the Vocabulista in Arabico (second half of the thirteenth century) and PEDRO DE ALCALA's Spanish-

Arabic vocabulary (second half of the fifteenth century) contain the word manâkh, meaning almanaque, calendario. Moreover the Arabic MS. 977 of the British Museum (3) contains a text beginning « Hadhâ manâkh. » — I quote the Latin translation — « Hoc est manâkh viri docti amplissimi ABÛ-L-ABBÂS AHMAD IBN AL-BANNÂ, (4) cujus ope cognoscitur, in quemnam diem annus Arabicus mensesque ejus cadant. » This manâkh is a regular almanac ! It is true the MS. is a late Mogrebbin MS. (eighteenth century) but that does not matter, inasmuch as we have independent proof of the use of the word manâkh in Spain, as early as the second half of the thirteenth century, in the sense of almanac. It is thus probable that the word almanac came to us through the Arabs of Spain. (See also the long article munâkh in R. DOZY's Supplément, vol. 2, 734, 1881.)

However manâkh can not be traced back to any Arabic root. What may be its origin ? A passage of EUSEBIOS (*De praep. Evang.*, III, 4) quoting PORPHYRY, alludes to astrologers whose names are found ἐν τοῖς ἀλμενιχιακοῖς(5). This is almost certainly an interpolation. But it suggested that the word manâkh might be of Coptic origin. My Harvard colleague, L. WIENER, has kindly consulted a number of dictionaries unintelligible to me, without being able to confirm this hypothesis. (6) In favor of the Egyptian origin, I may add that the word manâkh which is not found in every Arabic dictionary, is found in some Egyptian and Sudanese dictionaries, with the meaning of climate (JOSEPH CATAFAGO's English Arabic Dictionary, London, 1873. S. HILLELSON's Sudan-Arabic vocabulary, London, 1925).

Prince BONCOMPAGNI (7) had suggested that the word almanac was introduced through the title of the popular tables published by the Provencal Jew PROPHATIUS (JACOB BEN MACHIR IBN TIBBON) c. 1300. These tables have certainly helped to diffuse the word, but its use is earlier, as I have already shown. Even the Latin word almanac appears before that date. According to MURRAY's New English Dictionary (vol. I, 244, 1888) it was already used by ROGER BACON (c. 1267); but the following examples there quoted are much later : GIOVANNI VILLANI (c. 1345),

(3) Catalogus cod. mss. orient., nº 977, XI, p. 445, where the Arabic incipit may be found.

(4) This is the famous Moroccan mathematician IBN AL-BANNÂ' (second half of thirteenth century).

(5) According to SOPHOCLES's *Greek Lexicon* (Boston 1870, 118), this word is also found in PORPHYRY, Epistola ad Anebonem. It is *not* included in the newest edition of LIDDELL and SCOTT (1925).

(6) Professor WIENER would rather suggest a Sanskrit origin, via AL-BÎRÛNÎ. See « On the four measures of time called mâna » in al-BÎRÛNÎ's India (SACHAU's translation, vol. I, 353-55, 1888). This would explain the absence of manâkh from the earlier Arabic dictionaries and its apparition about the thirteenth century.

(7) Almanacco (*Giornale degli eruditi*, 3, 208, 1884.)

CHAUCER (1391); etc. M. STEINSCHNEIDER has given (8) a list of 16 Latin MSS. containing the word almanach in the title, one of which is dated 1231. If 1231 is the real date of this MS. (Tab. Arzachel, 1231; MS. Cambridge University, 1935), this is the earliest example of the word known to me.

That the words tacuinum and almanach were considered synonymous is proved by their simultaneous use, as in the following examples quoted by STEINSCHNEIDER : 1) in a fourteenth century MS. (Vatican 4087) one reads « scientia faciendi almanach i.e. tacuinum » ; 2) the Hebrew translation of GIOVANNI BIANCHINI's astronomical tables (second half of fifteenth century) contains both words; 3) the title of a work of ABRAHAM ZACUTO (second half of fifteenth century) contains also both words (ed. 1525).

To sum up, the two synonyms tacuinum and almanac are of Arabic origin. The first is simply the transcription of the good Arabic word taqwîm, meaning table (dispositio per tabellas); the second is derived from a bastard Arabic word manâkh, meaning climate, calendar. The almanac enigma is not completely solved, but it is reduced to finding the origin of the Arabic word manâkh.

Nov. 16, 1927. GEORGE SARTON.

(8) Über das word Almanach (*Bibliotheca Mathematica*, 1888, 13-16).

Reviews

(al-Jaḥiẓ). — *Three essays of* Abû 'Othman 'Amr ibn Baḥr al-Jâḥiẓ (d. 869). Edited from three manuscripts, with introductions by Joshua Finkel. 78 p. (all in Arabic). Cairo, Salafyah Press, 1344 (1926).

These three essays or epistles had been hitherto unpublished or badly published. J. Finkel has obtained better MSS. of them in the 'Azhar Mosque and in the Libraries of Taymûr Pasha and of Nûr al-dîn Bey in Cairo. By far the most important is the first, Al-radd 'alâ-l-Naṣârâ (29 p.), Reply to the Christians, which is indeed of considerable interest. It seems to have been written at the request of the 'Abbâsid caliph al-Mutawakkil, who undertook to persecute the unbelievers and heterodox in order to protect the Muslim state. Feeling ran especially high against the Christians, not a little because of their cultural superiority and of their high social standing. I have shown in my *Introduction* (e.g., vol. I, 549) how the Muslim renaissance was largely prepared by the Christians living in their midst. Al-Jâḥiẓ's treatise is of special value because of the indirect light which it throws upon this : his hatred of the Christians is an excellent confirmation of their prestige, social and intellectual; in the same way his relative toleration of the Jews is a witness of their relative insignificance. Al-Jâḥiẓ was essentially a man of letters and his jealousy was essentially an intellectual jealousy; his hatred of the Christians was fed by literary rancors. He himself knew no language but Arabic and he could not bear the overwhelming superiority which the polyglot Christians of Bagdad had over him. He does his best to undermine their prestige; he shows that the sources of their knowledge are neither Christian nor Byzantine, but Greek. The Christians, unlike the Jews, had increased their influence by mixed marriages. This introduces an interesting remark on the dangers of inbreeding (See also al-Jâḥiẓ's Kitâb al-ḥayawân, vol. I, 63-64, 71-72). He also denounces the Christian praise of celibacy and their occasional practice of castration. I am glad to hear that an English translation by J. Finkel of this bitter but most interesting pamphlet will soon be published in the *Journal of the American Oriental Society*.

A briefer mention of the two other epistles will suffice. The Akhlâq al-kuttâb, Characters of secretaries (12 p.) deals with the secretaries of some of the great Muslim leaders and is a condemnation of the whole

profession. The Risâlat al-qiyân (23 p.) deals with women singers, justifying the men who attend their entertainments; it contains very liberal views with regard to the casual relations between men and women, the wearing of veils, etc. This too is very striking but can not be discussed here.

These three treatises confirm our general opinion of AL-JÂḤIẒ : he was a very clever writer, original to the point of eccentricity, a man of immense learning but of little conscience, without principles or ideals except his literary standards, a good prototype of so many of the mercenary and unscrupulous writers of our own days. In spite of his egotism, his writings are exceedingly interesting in what they reveal and in what they hide or try to hide. If one forgets a few traits which are characteristic of his type and makes due allowance for his prejudices, they constitute an excellent mirror of his age.

GEORGE SARTON.

Carl Schoy (1877-1925). — *Die trigonometrischen Lehren des persischen Astronomen* ABŪ L-RAIḤÂN MUḤ. IBN AḤMAD AL-BÎRÛNÎ *dargestellt nach al-Qânûn al-Mas'ûdî*. Nach dem Tode des Verfassers herausgegeben von JULIUS RUSKA und HEINRICH WIELEITNER. XII + 108 p., 50 fig. Hannover, HEINZ LAFAIRE, 1927. (Mk. 16)

The premature death of our friend CARL SCHOY was peculiarly tragical. (1) Very few persons will ever be qualified to advance our knowledge of Muslim mathematics, for such qualification involves a very rare combination of qualities. One needs to be a mathematician and astronomer, have a good knowledge of Arabic or Persian or both, historical sense, palaeographical experience, and be ready, as for any worthwhile achievement, to spend one's time and energy without counting. SCHOY combined all these qualities, that is, he had gradually combined them at the cost of long studies and in spite of ill health and increasing discomfort. His first study on Arabic astronomy appeared in 1911 and during the following fifteen years he had produced a good many valuable papers. It is hardly necessary to insist upon this, as many of these papers appeared in *Isis*, both before and after his death. He left behind a large number of MSS. (see list in *Isis*, 9, 94), some ready for publication, others in various stages of completion. We all felt that SCHOY, if he had been granted live ten or twenty years longer, would have increased considerably our understanding of oriental mathematics and astronomy. Valuable as his early studies were, his knowledge and experience were continually in-

(1) See his biography by JULIUS RUSKA, with portrait and bibliography, in *Isis*, 9, 83-95, 1927.

creasing, and we felt that the very best that he could give us was still to come. But it was not to be !

I deeply regret that the present work, which was completed in the autumn of 1924, could not be published more promptly, for it would have induced me to give even more importance to AL-BÎRÛNÎ than I did in my *Introduction* (vol. 1, 707-09) and especially to deal at greater length with his mathematical achievements. SCHOY's study of AL-BÎRÛNÎ's trigonometry is derived from one of his main works, the astronomical encyclopaedia dedicated in 1030 to the Ghaznawid sultan MAS'ÛD and named after him, Al-qânûn al-Mas'ûdî. The best way to indicate its true signification is to quote the author's own words :

« ...Doch den Leser wird vor allem die Beantwortung der Fragen interessieren : Was bietet ihm die Trigonometrie des BÎRÛNÎ *inhaltlich*, und *in welcher Form* wird dieser Inhalt dargeboten? Auf die erste Frage ist zu antworten, dass sie, an ihrer Zeit gemessen, eine Leistung ersten Ranges darstellt, die auf jedem Blatt den überragenden und ganz selbständigen Autor erkennen lässt. Dem Kenner aber wird das viele Neue auffallen, das BÎRÛNÎ zum damaligen Bestand der Trigonometrie hinzufügte. Ich denke da z. B. an die Ableitung der 9-Eckseite, die geniale Berechnung von chord 40° und sin 1°, die Ermittlung von π, die Kriterien bei der Berechnung der Sinusse und der Schatten, die Einführung der 2. Differenzen, den mehrmaligen Versuch zur Verallgemeinerung einer Regel u.a. Ebenso ist die Anwendung der in der sphärischen Trigonometrie gewonnenen Regeln auf verschiedene Probleme der sphärischen Astronomie höchst originell. Da BÎRÛNÎ das reguläre Siebeneck nicht behandelt, so habe ich Lösungen dieser Aufgabe nach THÂBIT IBN QURRA und IBN AL-HAITHAM im Anhang gegeben. Hinsichtlich der Sinus- und Tangententafel ist das von BÎRÛNÎ Gebotene nur insofern bemerkenswert, als er den Radius gleich 1 setzt und 2 Differenzreihen gibt. Ich habe bei dieser Gelegenheit im Anhang die Sinus- und Schattentafel aus dem zîj des ULÛGH BEG (gest. 1449), die ich dem Berliner pers. Ms. 280 entnahm, bis zum Argument 45° dargestellt. Die Zeit muss jetzt als vergangen betrachtet werden, wo immer von arabisch-trigonometrischen Tafeln gesprochen wurde, während wir nur die dürftigsten kannten.

Was die Antwort auf die zweite Frage anbelangt, so ahnt der die Schwierigkeiten kaum, der nicht weiss, was es heisst, einen mathematischen BÎRÛNÎ-Texte in einer abendländischen Sprache darzustellen. Den meisten Lesern würde sicher eine freiere Übersetzung in möglichst glatten Deutsch, unter tunlichtster Anwendung der modernen Formalsprache, am ehesten zusagen. Gewiss, ein Standpunkt, der Vieles füt sich hat, und dem ich auch, wenn es angezeigt schien, Rechnung getragen habe. Andrerseits habe ich oft geglaubt, den eleganteren deutschen Ausdruck der möglichsten Wahrung der Originaltreue opfern zu sollen. Dies hatte seinen Grund zuweilen auch darin, dass ich mit der Entzifferung und dem Verständnis des arabischen Textes rang, wo mir selbst einiges nicht recht klar wurde. Auch weiss ja der historische Forscher so gut wie ich, dass die Lektüre eines älteren, auch nicht arabischen Textes eben kaum eine bequeme Lektüre ist. So habe ich denn auch, um dem Leser nicht alle Gedankenarbeit abzunehmen, nur im Anfang reichlicher erläuternde Zutaten und Beweise gegeben. In der Hauptsache richtete ich mein Augenmerk auf die Darbietung der Materie als solcher; wer aus ihrem Studium den vollen Nutzen ziehen will, muss sich in der älteren Trigonometrie und Mathematik bereits umgesehen haben. »

As indicated by the author, this posthumous book does not contain only a critical translation of the trigonometrical part of AL-BÎRÛNÎ's Qânûn (ten chapters of the third section) but some other relevant material for the sake of comparison : the construction of the regular heptagon by THÂBIT IBN QURRA and by IBN AL-HAITHAM; the table of sines, Baqîya jadwal al-jaib, and the table of tangents (Ẓill ma'kûs), both compiled by ULÛGH BEG, minute per minute. (For the latter's Catalogue of Stars, published by the Carnegie Institution, see *Isis*, 2, 413-15). The readers of *Isis* had already obtained some knowledge of this through SCHOY's Beiträge zur arabischen Trigonometrie, which appeared in Vol. V, 364-99, 1923, but here we are given the complete tables.

This work is a capital addition to our knowledge of Arabic trigonometry. It made me hope that we may have some day a complete English translation of AL-BÎRÛNÎ's Qânûn al-Mas'ûdî, even as we have of his two other fundamental works, the Athâr al-bâqiya and the Ta'rîkh al-Hind. It would be worth while also to investigate his minor works, for AL-BÎRÛNÎ was undoubtedly one the foremost scientists of mediaeval times. One would wish to know every side of this many sided personality and to be able to reconstruct it in its full size amidst the strange surroundings of the Ghaznawid court in far away Afghânistân. There dwelt also his great contemporary, AL-FIRDAWSÎ. Think of evoking these two heroes in such a background, — but the accomplishment of so great a task would require considerable preparation by many scholars and the devotion of at least a whole life... It is good to realize that there is so much to do, even if we can only hope for ourselves to outline the work. Our much lamented friend CARL SCHOY carried us a good deal forward. May he rest in peace !

GEORGE SARTON.

Algazel. — *Tahafot al-falasifat*. Texte Arabe établi et accompagné d'un sommaire latin et d'index. Par MAURICE BOUYGES, S. J. XXX + 448 p. Beyrouth. Imprimerie Catholique. 1927.

This is the second volume, but the first to appear, of an Arabic series called *Bibliotheca Arabica Scholasticorum*, published by the Jesuit Fathers at Beyrouth. It will be followed by the answer to this book written by AVERROES and by an edition of AL-GHAZZĀLĪ's *Maqâṣid al-falâsifa*, now in preparation. As to the present publication it may be said shortly that, with the partial exception of AL-GHAZZĀLĪ's *Durra*, in the edition of LUCIEN GAUTIER, it is the first book by that theologian to be edited after the western manner directly from the MSS with a full critical apparatus, running analysis and six elaborate indices. A translation is promised.

This is hardly the place to enter on a technical estimate of this edition from the point of view of the arabist. It is enough to say that Father BOUYGES has gone to work elaborately and conscientiously, making more

or less complete use of seventeen MSS. His indices which cover 67 pages are very full and useful, not only for this book but for the technical Arabic of philosophy and theology in general.

But what in the history of philosophy and theology is the place and importance of this book ? The Arabic writing Aristotelians had built up a philosophical system in which the attempt was made to explain the universe purely on the basis of reason. This is, of course, the essential task of philosophy. But these Arabic Aristotelians were quasi-Muslims and had to see to it that they did not lay themselves open to any public charge of unbelief. Their teaching and their writings were, therefore, kept as much as possible within a narrow circle of special students and were couched in a language not easily understood outside of these circles. Their teachings, however, did leak out and were one of the dissolvents working upon orthodox Islam and, in reality, on the essence of Islam itself. This was combatted from the Muslim side in different ways. The elaborate atomic system — of matter and of time — of the Mutakallims was built up. I have given an outline of this system in this Journal (9, pp. 326 ff., 1927), « Continuous re-creation and atomic time ». This makes all existence depend, from moment to moment, on the unconditioned will of Allah and reduces man to an automoton with consciousness as part of this automatism.

AL-GHAZZĀLĪ's theological position was quite different and his attack was, therefore, different. He had himself been through the whole range of scepticism down to the most absolute denial of the ordinary laws of thought. He had doubted and doubted his doubts. From that he had been rescued by a curious combination of pragmatism and a mystical experience. He had of his own experience discovered that the vision of the mystic was veridical and that the theological system built up on the basis of these experiences, when applied to life, worked. The human soul, he held, was a spiritual entity of the same nature as Allah ; it could reach Allah and be taught by him; and that was all we could know. The faculty of reason in man is sound in its own sphere but is essentially destructive outside of its sphere and cannot reach the ultimate nature of things or deal with the spitriual world. He himself under its guidance had touched the depths of scepticism and knew that it could, of and in itself, construct nothing.

So he brought this weapon of the detructive reason to bear upon the Aristotelian philosophers. They professed to prove everything by reason So he takes up one by one their fundamental propositions as to the origin and ultimate nature of the universe. They could not prove that it had existed from all eternity or would exist to all eternity; they could not show what time or motion were, they could not prove the existence of a maker for the world; they could not prove that Allah wasíOne or that he

had qualities, still less that he could not know particulars. They could not prove that the world had any cause at all or that anything has a cause. Reason cannot demonstrate the relation of cause and effect, it can only show first one thing and then another, but not that one causes the other. This is what we know now as the position of HUME. In the same way they can prove nothing about the nature of the human soul, for them it is only a bundle of phenomena.

The treatise is thus purely destructive — a destruction of rationalism by reason itself. It clears the way for religious experience to act as a proof of the reality of revelation and continuing divine guidance. Most ironically, AVERROES is known to Islam only by his answer to this treatise and AL-GHAZZĀLĪ is known to western philosophy only as an opponent of AVERROES. But the anonymous influence of AL-GHAZZĀLĪ is deep upon western theology and had a large part in moulding the system of THOMAS AQUINAS himself.

(*Hartford, Conn.*) DUNCAN B. MACDONALD.

Leonardo Olschki. — *Geschichte der neusprachlichen wissenschaftlichen Literatur.* III. Band : GALILEI und seine Zeit. VIII + 479 s. gr. in 8°. Halle (Saale), MAX NIEMEYER Verlag, 1927. Geb. 22,50 RM.

Dieser dritte Band des groszen Werkes aus der Feder des Heidelberger Romanisten ist, wenn auch gelegentlich auf die beiden vorangegangenen Bände (Bespr. *Isis*, 4, 209; 7, 128) hingewiesen wird, doch als ein vollständig selbständiges Buch über GALILEI zu betrachten. Dasz uns OLSCHKI mit seiner intimen Kenntnis auch der schönen und populärwissenschaftlichen italienischen Literatur etwas ganz besonderes bieten würde, war klar. Aber man konnte kaum vermuten, dasz er das Buch sosehr mit seinem Herzen schreiben würde.

Seiner bisherigen Einstellung gemäsz stellt OLSCHKI die technisch-praktische Seite der Ausbildung GALILEIS in den Vordergrund, wie sie etwas vorher durch den nicht vom Glück begünstigten Mathematiker TARTAGLIA verkörpert worden war, den OLSCHKI liebevoll gegenüber dem sicher unwahrhaften und gespreizten CARDANO herausstreicht. Aber GALILEIS Bild als des ernsten Forschers, der sich auf das Platonische Zählen, Wägen und Messen stützt, wird noch schärfer in den Konturen, indem es auf den Hintergrund des halb poetischen, halb phantastischen, jeder strengen Begrifflichkeit abholden Philosophen GIORDANO BRUNO gemalt wird. GALILEIS technisches Genie (er war auch ein vorzüglicher Zeichner und wollte in der Jugend Maler werden) erscheint als eigentliche Voraussetzung seiner wissenschaftlichen Versuche, durch die seine theoretische Originalität erst Richtung und Ausdruck erhielt. Aber das philosophische und wissenschaftliche Bild GALILEIS kann gar nicht von seinem literarischen getrennt werden, und dieses letztere Bild ist wohl überhaupt

noch nie gezeichnet worden. Wir sehen GALILEI emsig kritische Kommentare schreiben zum « Rasenden Roland », der sein Lieblingswerk war, und zum « Befreiten Jerusalem ». Wir erkennen, wie der ausdrucksvolle, oft zornige, Stil seiner Schriften, die mit Streit begannen und mit Streit endigten, sich herausbildete : « Von der italienischen Prosa des Barocks hat sich nur die GALILEIsche als lebensfähig erwiesen und zwar nicht nur als Ausdruck eines groszen Geistes, sondern als Vorbild des nationalen Stiles ».

Selbstverständlich erfährt man in diesem Buch auch das wichtigste über das Leben GALILEIS, und von allen seinen Schriften wird uns der Inhalt dargelegt. Aber das alles nicht so, als ob der Leser noch gar nichts wüszte, oder vielmehr, es wird nicht auf die Besprechung von Einzelheiten z. B. der Verurteilung durch die Inquisition oder auf eine Diskussion von bestimmten Ausführungen der Dialoghi eingegangen, sondern es ist überall den groszen Linien der Entwicklung nachgespürt, der Entwicklung der äusseren Verhältnisse im Leben GALILEIS und der inneren in seinem geistigen Fortschreiten. So versteht ein Mann wie OLSCHKI wohl, welch groszes Interesse die Kirche an der Bekämpfung des Coppernicanischen Systems haben muszte, und er sucht menschlich verständlich zu machen, wie der Papst URBAN VIII. aus einem Bewunderer GALILEIS zu seinem unerbittlichen Richter werden konnte. Dennoch kann er nicht ohne Bewegung und ohne Entrüstung das Vorgehen der Kirche und besonders URBANS gegen GALILEI schildern, in einer Frage, in der schlieszlich doch die Kirche nachgeben muszte. Und wenn P. DUHEM, an dem S. 245 (Fusznote) scharfe Kritik geübt wird, vor allem die Kontinuität der Entwicklung betonte und GALILEI wohl etwas zu sehr vom Mittelalter abhängen liesz, so übergeht OLSCHKI diesem Zusammenhang bis auf einige andeutende Worte ganz und stellt uns GALILEI nur als den unerschrockenen und unerbittlichen Kämpfer gegen das Alte vor Augen.

Die wissenschaftlich Stellung GALILEIS ist wohl am besten S. 164 charakterisiert durch die Worte : « Das rein materialistische Verfahren des Messens, Zählens und Wägens wird überwunden und fruchtbar gemacht durch die Erschaffung einer ideellen Realität, nach welcher die reale erst in ihren wesentlichen Erscheinungen erkannt werden kann, und die, wie PLATO zeigt, den Sinnenschein unwirksam macht ». Diese Kennzeichnung ist, wie auch die darauf folgenden Ausführungen, durchaus richtig; es ist aber später wieder mehr die Erfahrung und das Experiment als Grundlage der Erkenntnis in den Vordergrund geschoben, mehr als es wohl GALILEIS Empfinden selbst entsprach. In seinem Brief an CARCAVI vom 5. VI. 1637 hat er ausdrücklich betont, dasz die physikalischen Gesetze willkürliche (wir dürfen sagen : apriorische) natürlich an die Erfahrung anschlieszende Schöpfungen des Geistes sind, die erst nach ihrer intuitiven Konzeption durch das Experiment kontrolliert

werden. Das hat TORRICELLI in einem Brief an M. RICCI vom 10.II. 1646 noch stark unterstrichen. Auf diesen Sachverhalt wurde in neuester Zeit öfters hingewiesen.

Wie der Zusammenhang mit der Scholastik, so ist von OLSCHKI auch der Zusammenhang mit der modernen Wissenschaft, sagen wir gleich mit der Relativitätstheorie übergangen worden. Wenn die heutige Wissenschaft erklärt, man könne gar nicht unterscheiden, ob die Erde um die Sonne oder die Sonne um die Erde sich bewege, ja wenn diese Behauptung sogar auf die Rotation ausgedehnt wird, und man es für denkbar (vorstellbar ?) hält, dasz die Milliarden wimmelnder Sterne sich alle auf die Hundertstelsekunde genau in 24 Stunden um die Erde drehen, haben dann diejenigen recht, die sagen : Seht ihr, wie unnötig und sogar falsch der Kampf GALILEIS war und wieviel besser er getan hätte, wenn er dem Rate des Kardinals BELLARMIN und anderer gefolgt wäre, er solle doch seine Auffassungen als reine Hypothesen hinstellen ? OLSCHKI hat diese Frage wie gesagt nicht aufgeworfen; aber die Antwort steht bei ihm überall zu lesen, vielleicht besonders gut auf S. 117, wo es heiszt : « Als Gestalt und Persönlichkeit ist GALILEI das lebendige Symbol der nach allgemein gütligen Erkenntnissen strebenden, sich selbst überlassenen denkenden Menschheit geworden ». Wer dieses Buch gelesen hat, der weisz, dasz ohne GALILEIS schlieszlich doch erfolgreichen Kampf gegen hergebrachtes Buchwissen auch EINSTEIN und die Relativitätstheorie nicht gedacht werden können.

München. H. WIELEITNER.

Raymond Lenoir. — *Les historiens de l'esprit humain.* VII + 173 p., Paris, ALCAN, 1926. (12 fr.)

A ce joli et énigmatique titre, l'auteur a ajouté en sous titre les noms suivants : FONTENELLE, MARIVAUX, LORD BOLINGBROKE, VAUVENARGUES, LA METTRIE. Ces remarquables écrivains du 18ème siècle dont l'activité s'exerça dans des carrières bien différentes eurent cependant un caractère commun les imposant à l'attention de l'historien désireux de comprendre la conscience intellectuelle d'une époque où l'on pensa tant : ils furent «quelques uns de ces êtres qui eurent le privilège de retrouver partout de l'humain ». A chacun d'eux, R. LENOIR a consacré une étude extrèmement vivante qui donne au lecteur l'impression de pénétrer non les mobiles de leurs actions extérieures, ou les détails de leur œuvre écrite, mais la source psychologique de leur inspiration généreuse.

Parmi ces grands hommes qui eurent un droit égal à la reconnaissance de la postérité, deux d'entre eux jouèrent un rôle de premier plan dans le mouvement scientifique de leur temps et présentent pour *Isis* un intérêt particulier : et cependant FONTENELLE et LA METTRIE dont nous allons dire quelques mots ne furent pas et exclusivement des savants.

FONTENELLE, qui devint le Secrétaire perpétuel de l'Académie des sciences, et qui en cette qualité donna dans l'histoire annuelle des travaux de cette société, un résumé clair, lumineux, accessible au public cultivé, des mémoires souvent touffus, encombrés de détails accessoires et difficiles à lire de ses collègues spécialisés, physiciens, chimistes, naturalistes ou médecins, fut aussi un génial vulgarisateur, et son œuvre répandit même parmi les dames et les mondains le goût et la curiosité de la véritable science; mais les écrits plus proprement littéraires à tournure philosophique tels que les célèbres *Dialogues des morts* dont le ton alerte, spirituel et badin masquent très habilement la sérieuse et intelligente réflexion relèvent de la même inspiration. R. LENOIR montre cela d'une manière irréfutable, et prouve ainsi incidemment que l'on mutile l'histoire de la littérature quand on veut à toute force la séparer de l'histoire de la science.

LA METTRIE, dont les livres philosophiques scandalisèrent ses contemporains qui virent en lui un athée ou un matérialiste militant, fut un médecin praticien, un disciple de BOERHAAVE dont il traduisit certains ouvrages, en résuma d'autres et répandit en France leur influence, un chercheur de vérités spirituelles, une âme généreuse et sensible, géné plutôt qu'excité par l'atmosphère de polémique que soulevait ses ouvrages et qu'il ne recherchait pas. Il faut lire le captivant portrait que LENOIR, à l'aide d'une grande érudition, trace de ce personnage en général mal connu; peut-être regrettera-t-on qu'il soit si bref et demanderait-on encore quelque éclaircissement ou complèment; peut-être aussi voudrait-on voir quelques références au bas des pages permettant d'utiliser ou de compléter le trop court travail de l'auteur; mais le caractère même des esquisses qui composent le livre que nous analysons s'opposait sans doute à ce que LENOIR fit apparaître l'appareil scientifique qui forme l'armature de son travail; nous espérons qu'il voudra bien consacrer à LA METTRIE, qui le mérite, et sur lequel il a attiré notre attention, un travail plus long, plus complet, et qui nous instruirait beaucoup.

HÉLÈNE METZGER.

Henry Daudin. — *Études d'histoire des sciences naturelles*. I. De LINNÉ à JUSSIEU. Méthodes de la classification et idée de série en botanique et en zoologie (1740-1790). II + 264 p. II CUVIER et LAMARCK. Les classes zoologiques et l'idée de série animale (1790-1830). XIII + 460 p. + 338 p., Paris, FÉLIX ALCAN, 1926.

M. DAUDIN pense que la plupart des livres écrits par des naturalistes sur le passé des sciences naturelles ne satisfont guère aux exigences les plus généralement admises d'une méthode historique. Aussi voyons-nous avec intérêt, mais sans surprise, un agrégé de Philosophie, chargé de cours à la faculté des lettres de Bordeaux, entreprendre cette étude sur un sujet qui, dans l'histoire de la biologie, se prête parmi tous à des

recherches purement littéraires, sans rapport avec l'état actuel et pratique de la question. En effet, quoique nous ne connaissions guère d'historien de la chimie ou de la physique qui n'ait abordé ces sujets par l'entrée du laboratoire, nous voyons tous les jours plus d'historiens de l'évolution qui approchent cette question par la porte de la pensée. Les sciences naturelles par elles-mêmes, pour cette catégorie de savants, dont DAUDIN, ne sont que des outils d'ordre presque secondaire, utiles, certes, mais pas absolument indispensables. De tels philosophes emploient l'anatomie comparée et la taxonomie de façon analogue aux géologues qui étudient les fossiles comme points de repère dans la stratigraphie, mais qui ne prennent aucun intérêt réel à la paléontologie.

La période dans les sciences naturelles — fin du XVIIIe et début du XIXe siècle — qui fait l'objet spécial des études de DAUDIN est d'ailleurs particulièrement favorable à un traitement philosophique de ce genre. D'un côté elle suit, comme les tragédies antiques, les prescriptions d'espace et de temps, en ce qu'elle se déroule presque entièrement à Paris et que ses limites temporelles sont très nettes ; d'un autre côté le sujet lui-même est bien separé des études antérieures de GESNER, d'ALDROVANDI, de LINNÉ etc. et des recherches postérieures de DARWIN et de ses contemporains.

L'auteur se propose d'abord de « préciser, par une enquête sur le passé (de la zoologie et de la botanique), les termes dans lesquels le problème de la méthode naturelle », — autrement dit, de la représentation scientifique des affinités des animaux et des plantes, — se présente vers la fin du XVIIIe siècle aux botanistes et aux zoologistes ». Tel est le contenu du premier volume, sorte d'introduction générale aux deux tomes sur les classes zoologiques et l'idée de série animale à l'époque de LAMARCK et de CUVIER. Cette enquête commence par ARISTOTE, elle passe de là presque directement à quelques-uns des prédécesseurs les plus proches de LINNÉ, à LINNÉ lui-même, aux naturalistes de son époque et à ceux qui ont succédé à LINNÉ et précédé CUVIER, tels que LAMARCK (en botanique) et A. L. DE JUSSIEU.

Le plan du livre dépend du jeu de deux idées très distinctes et souvent antagonistes : 1. idée d'une classification « systématique » ou « méthodique » qui, procédant d'après des caractères déterminés, distribue un ensemble d'êtres donnés en fractions de plus en plus petites, toujours subordonnées, définies et circonscrites d'après des règles fixes; 2. idée d'une série « naturelle » qui relie les uns aux autres tous ces êtres par une suite continue de « rapports » indissolubles. Ces deux idées ont été employées pour des buts différents par le même naturaliste. Ainsi, A. L. DE JUSSIEU, qui règle la formation de groupes nouveaux sur une analyse comparative des groupes déjà établis, abandonne, quand le problème qui se pose à lui est celui de la formation des classes, la marche ascendante

qu'il a constamment suivie, pour passer des « composés » aux « plus simples », c'est-à-dire du général au particulier.

Ce premier volume passe en revue : 1. La classification à l'époque de LINNÉ: ses données et ses méthodes. 2. L'idée de série à l'époque de LINNÉ: ses antécédents et ses facteurs. 3. L'histoire naturelle après LINNÉ : les « méthodes » et la série. 4. Le problème de la « méthode naturelle » en botanique et en zoologie.

Le sujet de la partie principale de l'ouvrage, fruit de plus de vingt ans de travail, est de savoir quel était, au juste, vers 1860, le dergé de nouveauté de l'interprétation transformiste proposée par DARWIN, pour comprendre la signification des équivalents plus ou moins complets qu'on a cru souvent lui trouver chez des écrivains ou des savants du XVIIIe ou de la première partie du XIXe siècle; pour aider à juger, enfin, si les conceptions généralement admises auxquelles elle s'est substituée avaient, lors de son avènement, épuisé leur fécondité.

L'auteur expose la manière dont la classification des animaux, encore si rudimentaire chez LINNÉ et chez ses successeurs les plus proches, en est arrivée à se constituer telle qu'elle se présente, sous des traits assurés et, en très grande partie, définitifs, dans les dernières œuvres de CUVIER (*Règne animal*), de LAMARCK (*Histoire naturelle des animaux sans vertèbres*) ou dans les publications d'autres naturalistes, tels que LATREILLE, SAVIGNY ou DE BLAINVILLE, à l'époque de ces deux grands ouvrages.

Le premier plan de l'étude est occupé par CUVIER et LAMARCK, et l'auteur essaye de faire ressortir pour chacun d'eux une évaluation impartiale et exacte de l'originalité réelle et de l'action historique de ces deux grandes personnalités. Mais, tout en se proposant d'éclairer l'histoire de quelques notions d'une grande généralité et d'une portée théorique incontestable (« méthode naturelle », subordination des caractères et corrélations, progression et types de l'organisation), l'auteur ne se dispense pas d'analyser avec un certain détail et pour eux-mêmes les travaux proprement zoologiques auxquels elles ont dû de vivre et de produire scientifiquement.

Après une exposition de ce qu'était le Muséum et le travail scientifique en zoologie, exposition qui comprend une vue d'ensemble des naturalistes parisiens, de l'institution, de ses premiers professeurs, de GEORGES CUVIER et de sa technique, l'auteur nous donne un tableau descendant de la formation des classes, en commençant par les Vertébrés et en terminant par les « Zoophytes ». Sont ajoutées à cette partie quelques idées d'ordre général sur la théorie de la méthode et la formation des classes, telles que les caractères généraux de la réforme de la classification, le principe de la subordination des caractères, les lois de coexistence et unité de plan dans les premiers travaux de CUVIER, etc.

Une troisième partie, enfin, traite de la coordination des classes, sous

les deux sous-titres de « CUVIER : l'anatomie et les plans généraux de l'organisation » et « LAMARCK : la défense de la série et la genèse des êtres vivants ».

L'auteur conclut que « pendant tout le temps où elle s'est attachée surtout à établir une classification « naturelle », la zoologie a mis en œuvre l'idée-mère des théories de la descendance sans l'avoir adoptée... Quand elle a délibérément conçu, à partir de DARWIN, la recherche d'une classification positive comme identique, en son fond, à la détermination de la généalogie du monde vivant... elle a libéré, dans ce domaine, l'intuition scientifique des formules restrictives qui avaient si longtemps essayé de la discipliner, sans jamais y réussir de façon durable ».

Les trois tomes de DAUDIN constituent un excellent exposé de la période qu'il a si complètement étudiée. On ne peut s'empêcher de regretter, pourtant, que l'auteur n'ait pu joindre à ce résumé philosophique quelques données tirées de l'observation des faits, et n'ait pu écrire ce gros ouvrage dans un style plus attrayant pour l'homme de laboratoire.

(*New York*) RAOUL M. MAY.

Felix Klein. — *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert.* Teil II. Die Grundbegriffe der Invariantentheorie und ihr Eindringen in die mathematische Physik. Für den Druck bearbeitet von R. COURANT und ST. COHN-VOSSEN. x + 108 p. JULIUS SPRINGER, Berlin 1927. (Geb. RM. 13, 25).

Mit diesem Band sind die Vorlesungen KLEINS, deren ersten Band ich in *Isis*, 9, S. 447 besprochen habe, abgeschlossen. Leider ist dieser Band noch mehr ein Torso als der erste. Ein bewundernswerter Torso freilich ! Trotz der Neurasthenie, an der KLEIN seit Jahrzehnten gelitten hatte, hat er als 70 jähriger die neuen Theorien der allgemeinen Relativitätstheorie, die alle Mathematiker und Physiker bewegten, mit einem staunenswerten Fleisz durchgearbeitet. Nach Mitteilung der Herausgeber hat KLEIN 7 umfangreiche Aktenmappen darüber hinterlassen. Darunter sind 3 Kapitel, die hier zum Druck gebracht werden, aber auch eigentlich noch nicht druckfertig waren. Diese 3 Kapitel behandeln die mathematische Vorgeschichte der EINSTEINschen Theorie von den ersten Invariantentheoretischen Ansätzen an. Demzufolge handelt das erste Kapitel von den Grundbegriffen der linearen Invariantentheorie, das zweite von der speziellen Relativitätstheorie, das dritte von den Gruppen analytischer Punkttransformationen bei Zugrundelegung einer quadratischen Differentialform.

Das Wort « Vorgeschichte » musz man dabei mehr im Sinn einer « inneren Geschichte » nehmen, im Sinne der inneren Entwicklung der Theorien. Eigentlich historische Kapitel sind nur wenige und kurze eingeschoben. Insbesondere fehlt das persönliche Moment, das den ersten

Band so reizvoll machte, fast ganz. Aber es finden sich auch in diesem Bande verstreut schöne, allgemeine historische Wahrheiten, wie S. 70 ein Zitat aus der Antrittsrede von JACOBI über das langsame Wachsen der Wissenschaft, und S. 189 ein Ausspruch von KLEIN selbst über den Einflusz der zufälligen Tradition und Schulbildung auf die Entwicklung der Theorien.

Das vierte Kapitel sollte die allgemeine Relativitätstheorie behandeln, sowie die HAMILTONsche Mechanik unter Hervorhebung der Rolle der Theorie der Berührungstransformationen und der kontinuierlichen Gruppen. Verschiedene Entwürfe dazu lagen vor, aber keiner in einer Form, dasz er ohne grosze Ergänzungen hätte veröffentlicht werden könnnen. So haben die Herausgeber verzichten müssen, dieses Kapitel überhaupt zu bringen. Wenn sie mit Recht sagen, dasz es ja über die Relativitätstheorie selbst genügend Darstellungen gibt, so wird man doch bedauern, dasz dem letzten Werke KLEINs sozusagen der Giebel fehlt.

München) H. WIELEITNER.

Wilhelm Ostwald. — *Lebenslinien. Eine Selbstbiographie.* III. Teil : Gross-Bothen und die Welt, 1905-1927. XI + 487 S., 1 Bild. Berlin, KLASING u. Co., 1927.

Der dritte Band von OSTWALDS Memoirenwerk erfüllt reichlich die gesteigerten Erwartungen, welche die beiden ersten erregten. Hat die im zweiten Band (*Isis*, 10, 74-80) geschilderte Zeit den Gelehrten aus der engen baltischen Heimat in das deutsche und internationale wissenschaftliche Leben hinausgeführt, so tritt er nun aus dem Universitätsleben hervor und gibt sich, durch kein Amt gehindert, der theoretischen und praktischen Förderung allgemeiner Aufgaben hin, die in der Richtung seiner Ideale liegen. Ein sich stets erweiternder Wirkungskreis erhöht von Jahr zu Jahr seinen Einfluss auf das öffentliche Leben, bis der Ausbruch des Krieges diese Entwicklung zum jähen Stillstand bringt. Und nun folgt der vierte Satz dieser Lebenssymphonie : die Rückkehr aus dem « Lärm der Strasse » zur wissenschaftlichen Arbeit, jetzt im eigenen Heim und mit neuen, selbstgestellten, umfassenden Aufgaben, die zu grössten Erfolgen führen. Schliesslich, die endgiltige Zuwendung zu einem arbeitsreichen « Gartenfrieden » : ein Finale, in dem der Verfasser dieses jugendfrischen Buches sich als Gestalter seines Lebens zu einem Kunstwerk von imponierendem Schwunge zeigt.

Das nächste grosse Ereignis nach dem Bruch mit der Leipziger Universität ist OSTWALDS Berufung zum ersten deutsch-amerikanischen Austauschprofessor, veranlasst durch die Harvard-Universität. Er liest dort über Chemie und Naturphilosophie; die Eindrücke, die er empfängt, sind überaus freundliche und die, welche er hinterlässt, scheinen die

günstigsten zu sein — bis auf das Befremden, das er durch freimütige Äusserung atheistischer Gedankengänge erregt. Die Zahl der Einzelvorträge an verschiedenen Orten wächst ins Ungeheure, desgleichen die der ihm erwiesenen Ehrungen; erst in der Heimat ist wieder einige Erholung möglich. Als nächste grosse Aufgabe unternimmt er die Bearbeitung der Biographien grosser Forscher aus naturwissenschaftlichen, insbesondere energetischen Gesichtspunkten. Dem Buch « Grosse Männer », das sechs solche « Psychographien » (von DAVY, FARADAY, LIEBIG, J. R. MAYER, GERHARDT und HELMHOLTZ) und anschliessende allgemeine Gedanken bringt (*Isis*, 1, 124), ist ein grosser wissenschaftlicher und auch buchhändlerischer Erfolg beschieden. Die Einreihung der Forscher in zwei Typen, Romantiker und Klassiker, zurückgeführt auf grosse und geringe geistige Reaktionsgeschwindigkeit, ist sicherlich von dauerndem Wert und bildet einen Anfang zur Klassifikation der genialen Menschen. Wichtige Folgerungen zieht OSTWALD unter anderem für den Hochschulbetrieb. Die gewonnene Einsicht in die häufige verderbliche Wirkung der heutigen Mittelschulen macht ihn alsbald zu einem Vorkämpfer der Mittelschulreform, besonders wendet er sich gegen die Geltung der natürlichen, vor allem der alten Sprachen als wichtiger Bildungsmittel. Seine dahingehende öffentliche Tätigkeit hat der Reformbewegung eine neue Note gegeben und auch segensreiche Folgen gezeitigt — soweit solche bei der soziologischen Struktur unseren höheren Schulen möglich sind. Mit diesem Wirkungskreis hängt dann OSTWALDS Tätigkeit für die Annahme einer künstlichen internationalen Hilfssprache zusammen. Zuerst für Esperanto eintretend, vermag er dessen Verbreitung vornehmlich in den Vereinigten Staaten ausserordentlich zu fördern. Es folgt (1907) die Pariser Tagung der Delegation zur Schaffung einer internationalen Hilfssprache unter seinem Präsidium, die Schaffung des Ido und die bedauerliche Abspaltung der Esperantisten. — In Jahre 1910 lässt sich OSTWALD durch ERNST HAECKEL bewegen, das Präsidium des Monistenbundes, der grössten Freidenkervereinigung Deutschlands, anzunehmen. Durch seine Tätigkeit als Kongressleiter, Redner und Redakteur verleiht der Gelehrte dem Bunde einen starken Aufschwung. OSTWALDS « Monistische Sonntagspredigten » werden als Dokumente der Geisteskultur des frühen zwanzigsten Jahrhunderts dauernden Bestand haben. — Im Jahre 1911 gelingt OSTWALD eine seit langem angestrebte Gründung auf seinem früheren Fachgebiet, die des Internationalen Chemikerverbands, mit einem grossen Aufgabenkreis : Internationale Nomenklatur, Atomgewichte, Formeln und Zeichen, Literaturregister, gemeinsame Berichterstattung u.s.f. OSTWALD vermag auch den Verband für einige persönliche Lieblingsgedanken : Benützung einer künstlichen Weltsprache, normierte Formate für Drucksachen, zurgewinnen. Wer den ausserordentlichen Aufwand an Scharfsinn, Energie

und Geschick bei dieser Gründung betrachtet, wird es wohl bedauern, dass der Verband infolge des Krieges eine Gestalt angenommen hat, die den selbstlosen Absichten des Gründers völlig zuwiderläuft. — Die Frage der Vereinheitlichung der Formate wird aus OSTWALDS Mitarbeit an dem Institut « Die Brücke » verständlich, dessen Geschichte beweist, wie hingebend der Gelehrte sich Bestrebungen widmete, deren Gedanke von anderen ausging, jedoch in der Richtung seiner Ideale lag. Der Gedanke, der Zersplitterung infolge der modernen Arbeitsteilung entgegenzuwirken durch Arbeitsverbindung, durch das Bauen von « Brücken, welche eine Insel der Arbeit mit der anderen, zuletzt mit jeder verbinden », ist wahrhaftig ein richtiger und notwendiger und wird auch nach dem Scheitern dieses Unternehmens infolge der schlechten Eignung des Geschäftsführers wirksam bleiben.

Die Forschungsarbeit OSTWALDS in diesen Jahren ist der Naturphilosophie und der Soziologie zugewendet. Er setzt seine Bemühungen um ein System der Wissenschaften fort und geht darin über AUGUSTE COMTE hinaus. Das Buch « Moderne Naturphilosophie » bringt eine Bearbeitung der wichtigen Ordnungslehre.

In das Jahr 1910 fällt dann die wichtigste Ausdehnung energetischer Denkweise auf die Geisteswissenschaften, die Aufstellung des « Energetischen Imperativs », der als Grundlage der Ethik an Stelle von KANTS Kategorischem Imperativ treten sollte. Die Forderung « Vergeude keine Energie, verwerte sie », ist wohl berufen, eine Grundlage jeder wissenschaftlichen Ethik der Zukunft zu bilden.

Das Jahr 1914 bringt mit dem Ausbruch der Völkerkatastrophe, welche in Mitteleuropa nahezu jedes Einzelschicksal nachhaltig beienflusst hat, für OSTWALD das plötzliche Abreissen aller internationalen Beziehungen und das fast vollständige Aufhören der öffentlichen Betätigung. Vom Staate zu keiner Dienstleistung herangezogen, sieht sich der Nimmermüde gezwungen, seiner Arbeitskraft ein neues Feld zu schaffen. In dem Buch « Grosse Männer » hat er selbst darauf aufmerksam gemacht, wie mehrere Forscher auf der Höhe des Lebens ihrer Arbeit eine scharfe Wendung in ein anderes Gebiet gaben. Das gilt auch für OSTWALD selbst. Nun aber gibt der mehr als Sechzigjährige, durch die äusseren Umstände zu kräftiger Reaktion veranlasst, ein neues Beispiel : er wendet sich wieder der reinen Wissenschaft zu, jedoch nicht zur physikalischen Chemie zurück, sondern zu einer Aufgabe, die ihm eine Synthese nahezu aller seiner Anlagen und bisherigen Arbeitsgebiete gestattet : zur Farbenlehre und in späterer Folge zu zwei neuen, selbstgefundenen Wissenschaften : der Harmonielehre für die Farben und die Formen. In diesem Teile erhält das Buch auch eine neue Bedeutung für den Wissenschaftshistoriker: denn hier gibt OSTWALD eine gedrängte und doch in gewisser Hinsicht vollständige Einsicht in das Werden und Wachsen der Erkennt-

nisse im geistigen Neuland, ein seltenes Beispiel für die Psychologie der wissenschaftlichen Arbeit. Begreiflicherweise geraten OSTWALDS Lehren bei seiner radikalwissenschaftlichen Denkweise in Gegensatz zu den Ansichten vieler Künstler und besonders Kunstgelehrter, über deren « Scholastik » manches scharfe Wort fällt. OSTWALD geht auch da aus allen Kämpfen und Enttäuschungen mit ungebrochenem Optimismus hervor. Er macht sich an die wissenschaftliche Beobachtung der Alterserscheinungen; und im Begriffe, sich mehr der Malerei zu widmen, freut er sich « unbändig » auf sie und erhebt sich an dem Ewigkeitswert der « Partituren » seiner Bilder, die er in der selbstgeschaffenen Zeichenschrift herstellt. So klingt das bedeutende Werk würdig aus.

Wieder ist die grosse Zahl allgemeiner Ausblicke hervorzuheben, desgleichen die mit dem Auge und der Hand des Malers gearbeiteten Beschreibungen den persönlichen Erscheinung bedeutender Menschen. Von letzteren seien genannt : CHARLES ELIOT, WILLIAM JAMES, JOSIAH ROYCE, TH. W. RICHARDS, CHARLES NORTON, L. ZAMENHOF, RUDOLF DIESEL, GEORG HIRTH, KARL LINDE, ERNST HAECKEL, ERNEST SOLVAY, PH. A. GUYE, LUDWIG MOND, WILHELM FÖRSTER, BERTHA SUTTNER.

Das Namenregister zu den drei Bänden leistet unentbehrliche Dienste; ein Sachregister fehlt. Das Wegbleiben einer Bibliographie darf hoffentlich so gedeutet werden, dass noch zahlreiche Nummern für sie zu erwarten sind.

Brünn (*Tschechoslowakei*). ERNST BLOCH.

Isidore Lévy. — *Recherches sur les sources de la légende de* PYTHAGORE, 152 p. Biblioth. de l'École des hautes études, sc. religieuses, 42. ERNEST LEROUX, Paris, 1926. (30 fr.)

La légende de Pythagore de Grèce en Palestine. 352 p. Bibl. de l'École des hautes études, sc. histor. et philosoph., 252. H. CHAMPION, Paris 1927. (75 fr.)

De la critique des documents discordants échelonnés le long des dix siècles qui vont de XÉNOPHANE à PORPHYRE et JAMBLIQUE, l'auteur déduit que la légende « a pour base la tradition dévote de ceux qui vénèrent en PYTHAGORE l'homme-dieu venu sur terre pour enseigner une doctrine de salut et qui l'accrédita à force de miracles aux yeux des Grecs de Crotone et de Métaponte ». ARISTOTE a recueilli ce mythe où l'on « reconnait l'identité de PYTHAGORE et d'Apollon hyperboréen »; HÉRACLIDE l'a enjolivé; HERMIPPE, en le dénigrant, CALLIMAQUE, en se faisant le défenseur de certaines idées pythagoriciennes, ferment l'inventaire des ressources dont on dispose pour définir l'œuvre que lui avait consacrée HÉRACLIDE. A la fin du IVe S. et au commencement de la période hellénistique commence à se constituer une biographie d'apparence positive (ARISTOXÈNE, DICÉARQUE, TIMÉE, ANDROCYDE). Puis, vers la fin du

IIe S., le pythagorisme, profondément influencé par l'Académie, reprend une vitalité nouvelle, et il se constitue alors le Roman composite qui nous est connu par JAMBLIQUE et NICOMAQUE, par APOLLONIOS DE TYANE, LUCIEN DE SAMOSATE, PHILOSTRATE, et ATHANASE D'ALEXANDRIE, et aussi par la *Vie de Moïse* de PHILON, qui « est un calque partiel d'une vie de PYTHAGORE... qui a dû avoir cours parmi les pythagoriciens ou pythagorisants alexandrins du dernier siècle ptolémaïque. »

Dans le second vol. ci-dessus signalé, ISIDORE LÉVY essaie de reconstituer, d'après toutes les sources connues « le roman tardif de PYTHAGORE » à l'époque romaine (livre I); les Descentes aux enfers de LUCIEN et VIRGILE, éclairées par des fragments d'HÉRACLIDE, d'HIÉRONYME DE RHODES et d'HERMIPPE, lui permettent de reconstituer une Katabase de PYTHAGORE disparue des vies précédemment étudiées, et qui n'est pas sans lien avec la révolution miraculeuse qui accompagne son arrivée à Crotone (livre II); la vie de MOÏSE (ARTAPANOS, PHILON, JOSÈPHE), sa descente aux enfers, le document démotique datant vraisemblablement de la période ptolémaïque, et connu sous le titre de *Conte de Satmi Khamoïs* (GRIFFITH, MASPÉRO — à ce titre, LÉVY préfère celui de *Roman de Siosiri*) ne sont que des pastiches de la vie de PYTHAGORE (l. III) ; le judaïsme alexandrin, le pharisaïsme, et l'essénisme « offrent, comparés au mosaïsme biblique, des caractères nouveaux, signes de la conquête du monde juif par les conceptions dont la légende de PYTHAGORE fut l'expression narrative et le véhicule » (l. IV) ; enfin, dans le livre V, l'auteur cherche à établir que « l'Évangile dissimule sous un vêtement oriental le système de croyances qui ... captivait sur les rives grecques et latines de la Méditerranée les esprits les plus divers ». L'Évangile « a séduit le monde antique parce qu'il lui apportait, empreint du plus pénétrant charme exotique, un produit de la pensée grecque, héritière d'un lointain passé indo-européen ».

L. G.

Berthold Laufer. — *Insect-musicians and cricket champions of China.* 27 p., 12 plates in photogravure. Anthropology Leaflet, 22. Chicago, Field Museum of Natural History, 1927.

This very attractive booklet is another fruit of LAUFER's extraordinary learning, extraordinary because of its range and depth, and also by its application to unusual subjects. It is well known that the Chinese and Japanese fancy insects, and especially crickets, but very few people in the West have any idea of their amazing knowledge of these strange pets. For example, do the male crickets chirp to attract the attention of the females; is the chirping a mating call ? This is possible but far from certain. Now the Chinese have made an interesting contribution to this problem. « Of the many species of crickets used by them, the females are kept only of one, — the black tree-cricket (*Homoeogryllus*

japonicus), called by them kin chung (« *Golden Bell* », with reference to its sounds), as they assert that this is the only kind of cricket that requires the presence of the female to sing. » Another remark deserves to be quoted verbatim : « The relation of the Chinese to crickets and other insects presents one of their most striking characteristics and one of the most curious chapters of culture-historical development. In the primitive stages of life man took a keen interest in the animal world, and first of all, he closely observed and studied large mammals, and next to these, birds and fishes. A curious exception to this almost universal rule is presented by the ancient Chinese. In accordance with their training and the peculiar direction in which their imaginative and observational powers were led, they were more interested in the class of insects than in all other groups of animals combined; while mammals, least of all, attracted their attention. Their love of insects led them to observations and discoveries which still elicit our admiration. The curious life-history of the cicada was known to them in early times, and only a nation which had an innate sympathy with the smallest creatures of nature was able to penetrate into the mysterious habits of the silkworm and present the world with the discovery of silk. The cicada as an emblem of resurrection, the praying-mantis as a symbol of bravery, and many other insects play a prominent role in early religious and poetical conceptions as well as in art, as shown by their effigies in jade. In regard to mammals, birds, and fishes, Chinese terminology does not rise above the ordinary, but their nomenclature of insects is richer and more colorful than that of most languages. Not only do they have a distinct word or even several terms for every species found in their country, but also numerous poetic and local names for the many varieties of each species for which words are lacking in English and other tongues. » There are a number of Chinese works devoted to crickets, the earliest being the *Tsu chi king* written by KIA SE-TAO (first half of the thirteenth century). I hope that LAUFER may sometime give us an annotated translation of that work, with references to the later Chinese publications on the subject. No one is better prepared to do so than he is. The Chinese knew that the chirping is produced by the motion of the wings; they had developed elaborate rules with regard to the rearing and training of crickets, and to their medical care in many circumstances ! Their interest in cricket-fights is sufficiently proved by the fact that they are prepared to pay fantastic prices for champions (up to $ 100, the value of a good horse in China). LAUFER gives abundant details about Chinese cricket-lore and his text is illustrated with twelve beautiful plates.

G. S.

Louis Mercier. — ***La Chasse et les Sports chez les Arabes.*** Illustrations

de l'auteur d'après des miniatures orientales. 256 pp. Paris, MARCEL RIVIÈRE, 1927. (20 frs.).

Il y a quelques années, M. LOUIS MERCIER, Consul de France, nous a donné une excellente édition et traduction du traité d'hippiatrie d'Ibn Hudhail (fl. c. 1356). J'en ai rendu compte assez longuement dans *Isis* (8, 346-49; voir aussi 8, 541; 9, 504). Nos lecteurs savent donc que M. MERCIER, qui a d'ailleurs passé vingt ans de sa vie dans des pays musulmans, était admirablement outillé pour écrire le présent ouvrage. Celui-ci s'adresse à un public plus étendu que le précédent. C'est une equisse assez poussée de la chasse et des sports (y compris la danse) non pas chez les Arabes, mais plus exactement dans les pays d'Islam. Car comme l'auteur le remarque à la fin de son beau livre (p. 244) les Arabes ne sont pas des « sportsmen »; au contraire « ils sont nettement utilitaristes et fuient tout effort désintéressé. C'est pourquoi j'ai dû, dans cette étude montrer que le polo est jeu persan et hindou, que toute l'équitation dite arabe, jusque dans ses instruments mêmes, est persane et mameluke et ainsi de suite. Si bien que, dans le domaine du sport, comme dans tant d'autres, les Arabes bénéficient à nos yeux d'une réputation qu'ils doivent presque entièrement aux éléments non sémites incorporés à l'Islam. »

Il ne peut être question de résumer ici cet ouvrage si plein de faits et d'idées. Je me bornerai à noter quelques points qui m'ont spécialement intéressé.

Chevaux arabes. M. MERCIER reprend la thèse hardie de M. SALOMON REINACH, d'après qui le cheval barbe loin de devoir la vie au cheval arabe, existait et florissait des siècles avant lui et a participé à la formation de la race arabe en Orient (p. 231). « L'imprégnation de l'Espagne et du midi de la France par le sang barbe, commencée trois siècles avant le Christ — à ne considerer que les temps historiques — s'est poursuivie pendant toute l'occupation arabe, soit directement pour le premier de ces pays jusque vers le XV^e^ siècle, et de façon beaucoup moins continue et moins directe, pour le second » On sait que le cheval arabe est une création relativement récente; du temps du Prophète les chevaux étaient encore relativement peu nombreux. Les pedigrees des chevaux arabes sont des documents qui ne méritent aucune confiance (p. 186). Les Arabes négligent leurs chevaux d'une manière scandaleuse; ceux-ci doivent leurs qualités à une sélection naturelle assez dure et à la vie du désert, la vie au grand air, pleine de privations. Quant à la sélection artificielle, les éleveurs arabes la pratiquent assez stupidement, car ils sont dominés par tout espèces de superstitions.

Fauconnerie. « La technologie est presque entièrement persane, excepté en Maghreb, parce que c'est en effet en Perse que la chasse au vol a été le plus en honneur et poussée le plus loin. Il est vrai que le pays et les contrées les plus rapprochées de lui au nord jusqu'au Caucase, à l'est

jusqu'en Asie centrale, au sud-est jusqu'aux Indes, offraient et offrent toujours les meilleures variétés d'oiseaux de chasse. » (p. 82). Cependant l'auteur nous dit plus loin que les gerfauts d'Europe étaient fort recherchés dans les cours orientales. Les Musulmans avaient étudié les oiseaux de chasse d'une manière assez approfondie. « L'un des traités arabes de fauconnerie de la Bibliothèque Nationale consacre treize chapitres à la nosologie des maladies incurables du faucon et soixante-douze chapitres à la nosologie et à la thérapeutique de ses maladies curables ! » A ce propos je regrette beaucoup que M. MERCIER n'ait pas essayé de dater les traités de fauconnerie (et les autres traités de chasse) utilisés par lui, ni d'en identifier les auteurs. Voilà longtemps que je cherche en vain des renseignements à ce sujet pour le second volume de mon *Introduction*. Par exemple, la *Bibliotheca accipitraria* de JAMES EDMUND HARTING (Londres, 1891) cite plusieurs traités persans, arabes et turcs, mais il m'a été jusqu'ici impossible de déterminer leurs dates et leurs rapports. Une étude comparée de ces traités serait extrêmement intéressante.

Cruauté des Musulmans envers les animaux. L'auteur donne plusieurs exemples de cette cruauté qui nous est presque incompréhensible. Ainsi, l'emploi de vieilles chamelles lorsqu'on court l'autruche au fort de l'été (p. 145); on s'en sert comme de réservoirs a eau vivants; on leur coupe la langue après qu'elles ont bu; « elles ne peuvent alors régurgiter ni ruminer et il ne leur reste plus que quelques jours à vivre. Chaque soir, à la fin de la chasse, on en tue une pour prendre l'eau qu'elle a gardée et donner aux chevaux sa chair et son sang ! » Il est inutile de rappeler l'histoire infâme des chiens de Stamboul qui furent abandonnés sur une île déserte, et condamnés à se dévorer les uns les autres. D'autre part, quelques animaux sont protégés par des tabous. Par exemple, au Maroc, il existe un *ḥubs* ou *waqf* dont les revenus sont destinés à l'hospitalisation des cigognes malades ou blessées.

Jeux de balle (*kura*). D'après le regretté EDMUND DOUTTÉ, il y a trois manières de jouer la *kura* au Maroc méridional, et les deux premières correspondent respectivement aux ieux de *soule au pied* et de *soule à la crosse* de l'ancienne France. Le jeu de *soule au pied* était répandu surtout en Normandie et en Bretagne; les Anglais l'empruntèrent, dit-on, à l'époque de la guerre de cent ans et l'ont distribué depuis dans le monde entier sous le nom de *football*. Quant à la *soule à la crosse* les émigrants normands et bretons l'ont portée au Canada où elle est devenue le jeu national.

J'en ai dit assez pour montrer tout l'intérêt de ce livre érudit et délectable, dont l'attrait est encore augmenté par une série de dix belles planches reproduisant des miniatures orientales.

GEORGE SARTON.

A. N. Whitehead and Bertrand Russell. — *Principia Mathematica.*

Second edition. Vol. II, pp. xxi + 742. Vol. III, pp. 491. Cambridge, The University Press, 1927.

Volume I of the second edition of *Principia Mathematica* has been reviewed in this Journal (Vol. 8, 226-31, 1926) by Dr. H. M. SHEFFER. The text of volumes II and III remains unchanged, while desirable changes are indicated and new matter is presented in a new introduction and three appendices, all in volume I ; and I shall be mainly concerned with this new matter. Most of the difficulties which occur in *Principia Mathematica* are connected with differences of type among propositions, and it is to these difficulties that the new matter is chiefly devoted.

We may begin with a simple illustration of the way in which differences of type arise among propositions. Let ϕz denote any elementary property, and form the function $\phi x \vee \sim\phi x$. Then the proposition $(\phi,x).\ \phi x \vee \sim\phi x$ is a logical truth; and it is to be noted that this proposition itself is not among the values of ϕx, nor can the range of ϕx be increased so as to include this proposition (1). In precisely the same way, propositions such as $(\phi) : (\exists x).\ \phi x$ and $(\exists\phi) : (\exists x).\ \phi x$ cannot occur as values of elementary functions ; they are propositions of a higher type. Now we might be inclined to render the proposition just stated by « every proposition is true or false » ; but this cannot refer to propositions that are not elementary; it does not say, for example, that $(x) : (\exists\phi).\ \phi x$ is either true or false. In order to state the principle for propositions like this one, we seem to require a variable f, to denote propositions such as $(x) : (\exists\phi).\phi x$, so that we can say $(f).f \vee \sim f$. Again, since propositions like $(f).f \vee \sim f$ do not fall within the range of the variable f, we require a third proposition. The necessity for resolving propositions into types in this way would not be objectionable of it were not for the fact that, in some cases, we seem to require to speak, for example, of « all functions » of a given argument, without restrictions, and it would seem that no variable can denote functions in the required sense.

I wish to explain what is the chief novelty in principle which occurs in the new matter; but in order to do this, one further point must be noted. In the first edition, it was shown that every logical proposition can be derived from some matrix by generalization (2). A matrix does not contain any generalized constituents, but does contain variables, sometimes of various orders. Thus, $\phi x \vee \sim\phi x$ is a matrix, as is $f \vee \sim f$; and the foregoing propositions are derived by generalization from these matrices. Now it appears that the chief novelty of the new edition is the following. It is held that every logical proposition can be derived

(1) Cf. Vol. I, p. xxxiv.
(2) Cf. Vol. I* 9.

from some matrix by generalization, that matrix being such that it does not involve any constituents which take values themselves involving generalized constituents (3). Having constituents which take values involving generalized variables distinguishes the matrix f v $\sim f$ from the matrix ϕx v $\sim\phi x$; for, as we have seen, f denotes such propositions as (x): $(\exists\phi).\phi x$, whereas ϕx does not denote propositions containing apparent variables. It is held that we can dispense with such variables as f. I shall now try to indicate how this is possible.

We have seen that, *e. g.*, f v $\sim f$ has values such as

(1) $\quad (\phi) : (\exists x).\phi x : \text{v} : \sim(\phi) : (\exists y).\phi y,$

i. e. $\quad (\phi) : (\exists x).\phi x : \text{v} : (\exists\phi) : (y).\sim\phi y;$

so that the principle for these values is (f).f v $\sim$f. But it is also true that (1) is a logical consequence of $(\phi,x).\ \phi x$ v $\sim\phi x$. For

$(\phi) : (x).\phi x$ v $\sim\phi x : \supset\ : .(\phi) : .(y) : (\exists x).\phi x$ v $\sim\phi y : .\equiv :$

$(\phi) : (\exists x)\ \phi x.$ v $.(y) \sim\phi y : \supset : .(\phi) : .(\exists\psi): (\exists x)\ \phi x.$ v $.(y) \sim\psi y: .\equiv:$

$(\phi) : (\exists x).\ \phi x : \text{v} : (\exists\psi) : (y).\ \sim\psi y$; (4)

and it will be found that every value of f v $\sim$f is a logical consequence of (p).p v $\sim$p, where p denotes elementary propositions, so that (f).fv $\sim$f follows from (p).p v $\sim$p.It is important to note what is the principle by which, *e. g.*,

(2) $\quad (\phi,\psi,x) : \phi x \supset \psi x.$ v $.\phi x.\ \sim\psi x.$

is derived from (p).p v $\tilde{\text{o}}$p, since (2) is note a value of p v $\sim$p (5). In as much as p denotes any elementary proposition, it denotes any elementary proposition of the form $\phi x \supset \psi x$; and since the proposition holds for every p, it holds for every p of the form $\phi x \supset \psi x$; hence

$\quad (\phi x \supset \psi x) : \phi x \supset \psi x.$ v $. \phi x.\ \sim\psi x,$

i. e. $\quad (\phi,\ \psi,x) : \phi x \supset \psi x.$ v $.\phi x.\ \sim\psi x.$

The relation of p to $\phi x \supset \psi x$ is not the relation of function to value, in the ordinary sense ; the relation of p to $\phi x \supset \psi x$ is that of genus to species, whereas the relation of p to an elementary proposition is that of genus to instance. An instance denoted by a function is ***characterized by*** the function; whereas, a species is simply a more determinate function than its genus; it ***entails*** its genus, in the sense that whatever is characterized by the species must be characterized by the genus, *e. g.*, being an instance of $\phi x \supset \psi x$ entails being an instance of p.

It would seem,however,that although (f).f v$\sim$f follows from (p).p v$\sim$p, it might be impossible to *state* propositions of the order of (f).f v$\sim$f without employing variables which take values involving apparent variables. Mr. RUSSELL's theory, at this point, is somewhat puzzling : he

(3) Vol. I, p. xxix ff.
(4) p. xxixv ff.
(5) Cf. p. xxiii.

seems, at once, to wish to state propositions of this kind, and to maintain that no such propositions occur (6). The situation arises in the following way. Elementary functions of propositions that are not elementary are derived from elementary matrices; *e. g.*, $(x)\,\phi x.\ v\ .(\exists y)\sim\phi y$ is derived from $(x) : (\exists y).\phi x\ v\sim\phi y$. Now this derivation is held to be purely verbal : $(x)\,\phi x.v.(\exists y)\sim\phi y$ is held to be simply another sign for $(x): (\exists y).\ \phi xv\sim\phi y$. Accordingly, $(x)\ \phi x.\ v\ .(\exists y)\ \sim\phi y$ is not, in fact, an elementary function of propositions that are not elementary, but only appears to be; we are engaged here in a purely notational pantomine. In this view, I take it, it is not denied that there are propositions of the kind which $(x)\,\phi x.\ v.(\exists y)\sim\phi y$ appears to be, it being held simply that, in any case, nothing is said about such propositions. But if this is the case, propositions of an order higher than the second do not occur at all, because all such propositions result from definitions, alleged to be purely verbal ; and Mr. RUSSELL does not show how propositions of an order higher than the second could be stated without these definitions. In other words, it seems that a proposition such as $(\phi_2 z).\phi_2 a\ v\ \sim\phi_2 a$ could not be formulated at all without the definitions ; and if such propositions could not be formulated without the definitions, they could not be legimitately formulated with them, if the definitions are purely verbal. On the other hand, it is suggested that such propositions could be regarded as infinite conjunctions, and it appears that Mr. RUSSELL advances this view to meet the point I am considering (7) ; for when such variables as $\phi_2 z$ are introduced, there will be a proposition which is the defined equivalent of a proposition in which $\phi_2 z$ occurs if infinite conjunctions or disjunctions are resorted to. It must not, however, be concluded that when Mr. RUSSELL puts forward a definition, and holds it to be purely verbal, then the definition is certainly verbal; that these definitions are verbal is merely a theory concerning them, which is at least doubtful ; what is certain is that the definiens and the definiendum are logically equivalent, not that they are the same proposition.

It seems, however, that we are able to state analogues of propositions such as (f).f v∼f, without employing variables which take values containing apparent variables. Let t_1 denote any elementary matrix. Then, of course, we cannot say $(t_1).t_1\ v\sim t_1$, since elementary matrices are neither true nor false; but we can say « every value of every value of t_1 is true or false. » Let t_0 be any value of t_1. Then we might express this proposition by writing $(t_1) : (t_0).t_0\ v\ \breve{o}t_0$; and by assigning a value to t_1, let us say $\phi\ x \supset \psi x$, we obtain

(6) See p. xxiv, p. 130, and p. xxxiii ff.
(7) p. xxxiii.

$$(\phi x \supset \psi x) : \phi x \supset \psi x. \vee .\phi x. \sim\psi x,$$
$$\text{or} \qquad (\phi,\psi,x) : \phi x \supset \psi x. \vee .\phi x. \sim\psi x.$$

I think there is one point at which Mr. RUSSELL requires a principle of inference slightly wider than the one that he employs. The principle used is this: if we have $(\phi).F(\phi,\phi)$, we can infer (ϕ): $(\exists\psi).F(\phi,\psi)$ (8). This is dependent upon our being able to identify ϕ with ψ. But it is also clear that we can replace one occurrence of ϕ by $\sim\psi$, or by other elementary functions; and sometimes it is not sufficient to identify ψ with ϕ. As an example, consider the proposition

$$(\psi).\psi x \supset \psi y. \supset . (\phi).\phi y \supset \phi x,$$

which is equivalent to

$$(\phi) : . (\exists\psi) : \psi x. \sim\psi y. \vee . \sim\phi y \vee \phi x.$$

If we identify ψ with ϕ, we do not get a truth of logic; but it is sufficient to identify ψ with $\sim\phi$. Clearly, it is always sufficient to identify ψ with $\phi z \vee \sim\phi z$.

In this same connection, I think that Mr. RUSSELL allows an inference which is logically mistaken, and which is not required. From (1) (x).f(x,x) it is inferred, quite correctly, that (2) (x) : $(\exists y)$.f(x,y); but it is also inferred that (3) $(\exists y)$: (x).f(x,y). Now (1) may be rendered « it is false that there is an x such that $\sim$f(x,x) »; (2) may be rendered « it is false that there is an x such that $\sim(\exists y)$.f(x,y) »; (3) may be rendered « there is a y such that (x).f(x,y). » It is clear that (3) has instantial import, which (1) and (2) do not have, and thus that (3) does not follow from (1); and for the same reason, of course $(\exists x,y)$.f(x,y) does not follow from (1) (9).

There is one point of general significance, having to do with the kind of propositions which can properly be said to belong to logic, which I should like to discuss. It would be generally agreed, I think, that if a proposition is to be said to belong to logic, it must be such as can be certified on logical grounds alone, and must involve only logical constants and variables. But there seem to be two large classes of propositions, both of which satisfy this requirement; and the treatment of propositions in *Principia Mathematica* is held to be such as to involve propositions belonging to one of these classes only, namely propositions in extension. I wish to refer to propositions in intension in the traditional sense, not in the sense in which « intension » is used in *Principia Mathematica* ; and I wish to use the term extension in the traditional sense also, and to inquire how propositions in intension are related to the propositions which occur in the *Principia*.

Mr. RUSSELL holds that universal propositions are to be interpreted as factual universals, *i. e.* as propositions in extension. It is held that

(8) p. xli.
(9) Cf. p. xxiv.

$\vdash$. fx is to be understood to mean $\vdash$. (x).fx, and that $\vdash$. fp is to be understood to mean $\vdash$. (p).fp (10), and that (x).fx may be rendered « all propositions of the assemblage indicated by fx are true » (11). I think it possible that some confusion may arise with regard to the precise force of a universal proposition. The force of a universal proposition is always negative, in the sense that it merely denies the occurrence of a certain kind of case. Thus, (x).fx is rendered more naturally by « it is false that there is a value of fx which is false » ; (x).fx neither asserts nor entails that there are values of fx, and for this reason it is a mistake to hold that the force of fx is given by the joint assertion of its values.

Now I think it relevant to inquire, with regard to the propositions which Mr. RUSSELL asserts, as for example propositions of the form $(x).\phi x \vee \sim\phi x$, how he knows these propositions to be true. (Strictly, of course, this is a purely epistemic question.) A proposition of the form $(x).\phi x \vee \sim\phi x$ is to be rendered «it is false that there is a value of $\phi x \vee \sim\phi x$ which is false.» We are not acquainted with all of the values of $\phi x \vee \sim\phi x$, nor, if we were, would it follow that we are acquainted with the fact that they are all. It seems quite certain that we know the proposition (1) « it is false that there is a value of $\phi x \vee \sim\phi x$ which is false » because we know a stronger proposition, namely (2) « there cannot be a value of $\phi x \vee \sim\phi x$ which is false. » (A proposition p_1 is stronger than a proposition p_2, if p_1 entails p_2, and p_2 does not entail p_1.) (2) is stronger than (1), and appears to be the only ground we have for asserting (1); for it does not at all follow from « it is false that there is a value of fx which is false » that there cannot be such a value ; it does not follow, for example, from « no men are immortal » that there cannot be immortal men. Now (2) is true on logical grounds, as well as (1), if (2) is our ground for asserting (1), and (1) is known to be true on logical grounds. However difficult it may be to give a satisfactory account of propositions in intension, such as (2), it seems quite clear that, in some sense, these propositions « transcend the actual » ; we know that (1) holds, because, from (2), we know that (1) would have to hold, no matter what might be the case, and thus that it *does* hold. But to say that (1) *does* hold (which is all that is said in *Principia Mathematica*, according to Mr. RUSSELL's view) is not in the least to say that (1) would have to hold, no matter what. We are, then, in this peculiar situation: Mr. RUSSELL asserts certain propositions in extension, which are true for logical reasons ; there are other, stronger propositions (in intension), true on logical grounds, which he never asserts ; while, at the same time, his knowledge of these

(10) p. xiii.
(11) p. xx.

stronger propositions seems to be his only ground for asserting the weaker propositions which he does assert. C. H. LANGFORD.

Charles Singer. — *The herbal in antiquity. Journal of Hellenic Studies*, vol. 47, 52 p., 10 colored plates and 46 figures in the text, London 1927.

Our readers will be glad to hear that SINGER's memoir on herbals, for which he had been accumulating material for a considerable time, has finally appeared. This very clear survey is not restricted to antiquity as the title indicates; it includes the Middle Ages as well and enables us to trace the main tradition in the West down to the incunabula. This paper is especially valuable from the iconographical point of view, for it contains a large number of illustrations which can easily be compared and thus provide concrete images of the botanical tradition. For example, there are admirable colored reproductions of the plants figured in the Johnson papyrus (c. 400 A. D.,) in the Cotton Vitellius C III (Anglo-Saxon work of c. 1050), in two MSS. of Apuleius-Dioscorides written in Germany c. 1200, in a herbal written at Bury St. Edmunds c. 1120, in two Anglo Norman MSS. of the thirteenth century. Many of the text figures reproduce in outline illustrations of the famous Juliana Anicia Codex of Vienna (By the way, in Vol. I of my *Introduction* I have spoken two or three times of this MS. as being in Venice, because I labored under the impression that it was one of the MSS. which Austria had been obliged to surrender to Italy after the war).

This memoir is so important that I think it worth while to reprint the author's conclusions. I do this chiefly for the sake of botanists who have no easy or continuous access to the *Journal of Hellenic Studies*.

« 1) There were no Greek herbals earlier than the fourth century B. C.

2) The earliest herbal we know of is that of DIOKLES, c. 350 B. C. The earliest herbal which survives is the 9th book of the *Historia plantarum* of THEOPHRASTUS. It is a collection from other herbals, and is not by THEOPHRASTUS. It is probably of Alexandrian date, say about 250 B. C.

3) Of other Alexandrian herbals we have few records. None has survived. The work, however, of NIKANDER (c. 200 B. C.) has come down to us. We have rough figures of an illustrated copy of NIKANDER which goes back to a classical original.

4) KRATEUAS (c. 75 B. C.) of Pontus was the first to prepare a herbal with illustrations drawn from the objects. He omitted the description of plants and fixed the form of the herbal. He is the father of botanical illustration.

5) We can restore part of the herbal of KRATEUAS, both as to text and figures, from the Juliana Anicia Codex of A.D. 512, aided by hints from a new illustrated papyrus herbal of about A.D. 400.

6) There were several Greek herbals written in the first century of the Christian era. Among them were those of PAMPHILOS of Rome, part of whose work *On Plants* survives : of MENECRATES, physician to the Emperor TIBERIUS, who originated Diachylon plaster; of ANDROMACHUS, who invented a complex antidote or « mithridate » that persisted into modern times; of SERVILIUS DAMOCRATES, of whose herbal we have considerable fragments, and of the army surgeon DIOSKURIDES of Anazarba.

7) The work of DIOSKURIDES has survived entire. It is the most influential herbal ever written, but was based on earlier herbals. There are Dioskuridean names in modern botany and a number of Dioskuridean drugs in modern medicine.

8) At an early date the text of DIOSKURIDES was associated with plant pictures, some from KRATEUAS, some from other sources. It was associated also with a list of synonyms in various languages. The resulting combination was the main herbal in use in the Byzantine period. Many MSS. have survived, and their line can be traced down to modern times. The finest and earliest is the Juliana Anicia, with many figures in the Krateuan tradition. An important element in the Dioskuridean texts that have come down to us is the list of synonyms derived from languages, many of which are now unknown.

9) Of Greek herbals after DIOSKURIDES only the *De simplicibus* of GALEN (c. A.D. 180) was of importance. ORIBASIOS borrows from it for his *Synagoge* of about A.D. 400. The herbal works of DIOSKURIDES, GALEN and ORIBASIOS were all early translated into Latin.

10) There is a Greek illustrated papyrus herbal of about A.D. 400 which we designate the *Johnson Papyrus*. It has figures of «Symphyton» and «Phlommos». It is important because — *a*) It is our earliest original of a herbal the general form of which is of the KRATEUAS and APULEIUS type. *b*) Though in Greek, its drawings are of a different tradition to that of other Greek herbals and different from the drawings of KRATEUAS. *c*) Its two drawings are stiff and conventional and strongly resemble those of certain early Latin APULEIUS herbals, with two of which they can be identified. *d*) Remnants of the tradition represented by the figures of the Papyrus herbal persisted to modern times among the Latins.

11) The earliest native Latin herbals were saturated with Magic. PLINY's herbal (c. A.D. 60) is Greek material. We have also herbals written in Latin in the third and fourth centuries A.D.

12) DIOSKURIDES was translated into Latin in the sixth century. Two distinct versions are available, designated as *Dioscorides Lombardus* and *Dioscorides vulgaris*. The illustrations to *Lombardus* are peculiar. The *vulgaris* type includes nearly all the vast number of MSS. Many of the illustrations of this main line were doubtless of the Krateuan tradition. The illustrations have become greatly modified in transit. An interesting

branch of the Latin herbal is the Anglo-Saxon version made about A.D. 1000. *Dioscorides vulgaris* is usually associated with other texts, notably that of «Apuleius». The *vulgaris* carried over with it from the Greek the ancient synonym lists.

13) The most important Latin herbal has the name of APULEIUS attached to it. It early became associated with synonyms similar to those of DIOSKURIDES. Two of these synonyms occur as titles in the Papyrus herbal. «Apuleius» became associated with other material, notably with the Latin Dioscorides. The MSS. of APULEIUS are exceedingly numerous. Their line can be traced into modern times. The Anglo-Saxon branch is again of peculiar interest. The earliest naturalistic plant-drawings are in a MS. of Apuleius-Dioscorides written in England (c. 1120). The tradition of the Apuleian illustrations can be traced continuously to the time of printing, and the first printed herbal has figures like those of an Anglo-Saxon herbal.

14) In the earliest MS. of APULEIUS (c. 650) are two figures that can be identified with those of the Johnson Papyrus (c. 400). The Apuleius and Papyrus herbals agree in general form. Apuleius is a translation of a Greek original similar to that of the Papyrus. Its illustrations have thus a different tradition from those of the Latin *Dioscorides vulgaris* or *Lombardus*.

15) «Apuleius», *Dioscorides vulgaris*, *Dioscorides de herbis femininis*, are all constantly found associated and intermixed with certain other texts. Among these other texts is a set of pagan invocations to the earth goddess which recall the conditions under which the herbal first came into being.

16) DIOSKURIDES became popular in the Arabic East as it did in the Greek and Latin West. His work was translated into Arabic about 854 at Baghdad. In 951 a new Arabic version was prepared at Cordova. About 1250, a Syriac translation with illustrations was prepared and an Arabic version again from that. A number of Arabic MSS. of DIOSKURIDES are known. The descent neither of the Arabic texts nor of the illustrations that accompany some of them has been traced, but the illustrations have no clear relationship to those of the classical line.

17) The classical tradition persisted to the days of printing. It then became diluted with new material, but was not wholly destroyed. Modern herbalists still practise the ancient herbal lore. The story of the herbal can thus be told almost continuously from the fourth century B. C. »

G. S.

Charles Singer. — The Evolution of Anatomy : A Short History of Anatomical and Physiological Discovery to HARVEY : Being the Substance of the Fitzpatrick Lectures Delivered at *The Royal College of Physicians of London* in the years 1923 and 1924. London, KEGAN

PAUL, TRENCH, TRUBNER and Co., Ltd. ; New York, ALFRED A. KNOPF, 1925, XII + 209 p., with XXII pl. and 117 fig. in the text. $ 4.

How may one do justice to this magnificent study ? It may be read in a couple of hours, but its inspiration should be lasting. It is an ideal treatment of the historical development of a science, brief, authoritative, brilliant in interpretation, delightfully written, comprehensive, judicious, superbly illustrated, and attractively published.

Dr. SINGER states his point of view admirably :

« In composing this short History of Anatomy I have been concerned to keep it within compassable limits. Unprogressive anatomical movements and periods are therefore but lightly touched upon, attention being concentrated on the line of definite advance. It appeared both unnecessary and undesirable to make any division between Physiology and Anatomy, at least in the period under consideration. As the narrative of the little book ends early in the seventeenth century, no mention has been made of the beginnings of Iatrochemistry in the persons of PARACELSUS, VAN HELMONT, and their followers, since the movement they represent did not become important until the second part of the seventeenth century. The volume closes with HARVEY. The new physiological movement, together with later anatomical developments will be treated in a separate work.

« Vivitur ingenio, caetera mortis erunt, *it is his genius that yet walks the earth* ; *all else of him may go down into silence*, is the motto which VESALIUS himself has chosen for the most beautiful of all his figures. Let him be taken at his word : I have sought to treat him and the great men who went before and after him as they would be treated. Prompted alike by personal inclination, by necessity for brevity, and by the suggestion of VESALIUS I have considered only the actual contributions to knowledge that these men have made, seeking to treat the History of Anatomy as a secular conversation between great minds, a debate of men of genius continued through the ages. If the writing of History cannot establish such continuity of ideas it can work nothing effectual... »

There have been several recent studies of the development of anatomy, chief among which are LUDVIG CHOULANT's great work on the history and bibliography of anatomic illustration, translated into English by the lamented Dr. MORTIMOR FRANK of Chicago (*Isis*, 4,357) and CHIEVITZ's nicely illustrated work unfortunately only available in Norwegian. Dr. SINGERS' effort surpasses anything that has yet appeared on the subject, although the bibliophile may miss the interesting little notes on the books themselves that so attract him.

Concerned chiefly with the development of gross anatomy, the study fittingly closes with HARVEY. As Dr. SINGER hints, after this period, chemistry transformed physiology, and the microscope ushered in a new era. Details of personal life have been deliberately omitted, but many stimuli remain for the reader to investigate these for himself. Thus, the significance of MICHAEL SERVETUS (1511-53) is covered in a short paragraph, but a hint regarding his tragic story, and a plate of

the Roche statue near Geneva serve nicely to call attention to one of the most dramatic incidents in history.

The scope of the effort is given by a list of its contents :

I. The Greeks to 50 B. C. (The Prescientific Stage; The Schools of Sicily, Ionia, and Cos, 550 B. C. — 400 B. C.; The Early Athenian Period. PLATO, DIOCLES, 400 B. C. — 350 B. C.; The Later Athenian Period. ARISTOTLE, THEOPHRASTUS, 350 B. C. — 290 B.C. Aristotelian Philosophy in its bearing on anatomical thought ; The Great Alexandrians, 300 B. C. — 250 B. C. (*a*) HEROPHILUS, Father of Anatomy (*b*) ERASISTRATUS, Father of Physiology ; Decline of the Alexandrian School, 250 B.C — 50 B. C.; Human Vivisection at Alexandria; The Alexandrian Anatomists and the Wisdom Literature.)

II. The Empire and the Dark Ages. 50 B. C. — A. D. 1050. (The Beginnings of Anatomy at Rome, 50 B. C. — A. D. 50; Latin Anatomical Literature ; Greek Anatomical Writers of the Early Empire. RUFUS SORANUS. A. D. 50 — 150 ; GALEN the « Prince of Physicians » 150—200; GALEN's Anatomical Philosophy ; GALEN's Anatomical Achievement; GALEN's Physiological System ; The Dark Ages, 200 — 1050.)

III. The Middle Ages and Renaissance. 1050 — 1543. (The Translators from the Arabic, 1050 — 1250; The Rise of the Universities. The Bologna School ; The Beginnings of Dissection. 1250 — 1300; MONDINO, The Restorer of Anatomy. 1300-1325 ; Mediaeval Anatomical Nomenclature Anatomical Knowledge of MONDINO ; The Later Middle Ages. 1325-1500; Naturalism in Art. LEONARDO DA VINCI. 1450- 1550; The First Anatomies printed with figures. BERENGAR, DRYANDER, ESTIENNE, CANANO. 1490-1545 ; The Humanists. BENEDETTI, LINACRE, MONTANUS, GUNTHER, SYLVIUS. 1450-1550).

IV. Modern Times to HARVEY. 1543-1628. (VESALIUS, the Reformer of Anatomy. 1514-1564 ; Threefold Character of VESALIUS : Artist, Humanist, Naturalist ; The Supply of Anatomical Material in the Fifteenh and Sixteenth Centuries ; The Seven Books of the *Fabrica* of VESALIUS 1543 ; EUSTACHIUS, Rival of VESALIUS. 1550-1574 The Followers of VESALIUS. COLUMBUS, FALLOPIUS, ARANTIUS 1550-1590 ; The Early Comparative Anatomists. VESALIUS, BELON, RONDELET, COITER, RUINI. 1540-1600 ; FABRICIUS. 1590-1610; The Last of the Great Paduans. CASSERIUS, SPIGELIUS. 1600-1630; Anatomy beyond the Alps, Switzer and, Holland, Denmark, France, Germany. 1590-1630; The Beginnings of Anatomy in England (*a*) The Middle Ages (*b*) The Renaissance ; The Work of WILLIAM HARVEY. 1628 ; Epilogue.) A Vesalian Atlas (of 20 pages) and an Index of Personal Names.

The outstanding features of the book are a matter for individual choice. The treatment of ARISTOTLE (384-322 B. C.) and GALEN (A. D. 129-99) is exhaustive but extraordinarily compact. Diagrams have been employed

with skill to illustrate difficult points, as in the discussion of GALEN's interpretation of the functions of the recurrent laryngeal nerves, and his general physiological system. The origins of scientific dissection in post-mortem examinations in cases of suspected poisoning in the thirteenth century, and its official recognition in Bologna in 1405, together with the effects of the famous Papal Bull of 1300, are briefly but adequately discussed. The startling effect on anatomy of the development of naturalism in art, under DA VINCI (1452-1519), DÜRER (1471-1528), MICHELANGELO (1475-1564) and RAPHAEL (1483-1531), is well illustrated. MONDINO (*c.* 1770-1326) has been exhaustively studied by Dr. SINGER in the first and second volumes of the superb *Monumenta Medica*, published by R. LIER in Florence under the editorship of Dr. SIGERIST (*Isis*, 8, 350). Likewise CANANO (1515-79) has been made the subject of a delightful study by Drs. CUSHING and STREETER in the fourth volume of the same series (*Isis*, 9, 433-4).

The artist in Dr. SINGER has openly been captured by VESALIUS. With a wealth of splendid illustration from the *Fabrica*, Dr. SINGER treats his hero gloriously, but not without justice to his rivals and followers. Of EUSTACHIUS (1520-1574), for example, it is pointed out that his copperplate of the sympathetic system and base of the brain is one of the clearest and best ever produced.

The section on comparative anatomy is valuable because it treats of a phase of the subject usually neglected. The scientific aspects of HARVEY's efforts are carefully outlined, and with an epilogue relating to the Paduan tradition in English anatomy, the text closes. May the sequel, treating of the developments of anatomy and physiology since HARVEY soon follow !

(*Univ. of California*) CHAUNCEY D. LEAKE

Raoul et Marg. Béclard d'Harcourt. — *La Musique des Incas et ses survivances*, vol. 1, Texte, 8 + 576 pp.; vol. 2, 23 pp. + 39 plates. Paris, PAUL GEUTHNER, 1925.

This study of popular music in the ancient empire of the Incas is mainly an account of the music as now found in Ecuador, Peru and a part of Bolivia. The authors have a good background for writing an authoritative volume ; they collaborated in two volumes published in 1924 : *La Céramique Ancienne du Pérou* and *Les Tissus Indiens du Vieux Perou*. In 1923 Madame D'HARCOURT published at Milan a volume entitled : *Mélodies Populaires Indiennes* (*Equateur*, *Pérou*, *Bolivie*), and in 1925 appeared at Paris Monsieur D'HARCOURT's volume *L'Amérique avant Colomb*. Furthermore they contributed jointly :« La musique dans la sierra andine de la Paz à Quito », *Journal de la Société des Americanistes*, Paris, vol. 12, 1920 ; and « La Musique indienne chez les anciens civilisés

d'Amérique » in *Encyclopédie de la Musique*, vol. 5, Paris, 1922. The general literature of the subject is rather extensive; 230 titles are given in the Bibliography, pages 545-556.

The work under review is divided into four parts. The first part (pp. 2-87) treats of the Instruments ; the second part (pp. 81-124) of Fetes and Dances ; the third part (pp. 125-230) deals with Musical Folklore ; and the fourth part (pp. 231-544) is devoted to Poetical and Musical Settings.

The Incan Empire was developed to a very high degree of efficience during the five hundred years preceding the arrival of the Spaniards, and the conquering of Peru, in the sixteenth century. Incan history and tradition were preserved by their bards, and accurate statistics were kept by means of knotted cords or quipu.

In order to study the music of the Incas and of their direct descendants the authors did not have recourse merely to documents since Indian musical notation never existed. But they sought to learn all that they could regarding their musical instruments which would suggest the extent to which the ancient empire's musical art had been developed. Such a study may be made partly by written and printed documents, and ceramics, but better still by the examination of the instruments themselves. A study of this kind in Peru is more fruitful than in any other country. Due to the dryness of the climate, no region, save Egypt in the old world, has preserved intact so many objects illustrative of earlier civilizations.

The ancient instruments are of three kinds : instruments of percussion (such as rattle, bells, gong, cymbals, drums, xylophones) ; wind instruments (horns, pan's pipes (*syrinx*), vertical flute, flageolet, ocarina, whistle); and string instruments (musical arc, harp, mandolin, and guitar). No stringed instruments were known before the arrival of the Spaniards. Apart from instruments of percussion and horns pre-Columbus America employed as instruments, truly worthy of the name, only flutes, even though flageolets were also known. The syrinx was brought to a degree of perfection unknown in the Old World. The instruments suggest that the pentatonic scale was fundamental. The authors believe that from the musical point of view the ancient Peruvians led all American peoples.

The second part of our work indicates the ancient association of music and the dance with calendrical and other feasts, official ceremonials, rituals, and funerals. Hardly anything is to be found descriptive of domestic music, such as the love song and the pastoral, in which were expressed the intimate thoughts of the Incas, « ces peuples les plus artistes de l'Amérique ».

The third part, dealing with musical folk lore, is exceptionally interes-

ting. The characteristic sadness of the songs is notable. The pure Indian melodies of the Incas were based on the four pentatonic modes A, B, C, and D. The crossing of races in Peru gave birth to new music and further mixed modes. As to rhythm the binary is most common, the ternary is generally slow, while 4/4 movements are rare. Mixed rhythms are also met with and the rhythms of accompaniments such as by clapping the hands, or by instruments of percussion. The popular music now includes : religious chants (survival of the ancient chant), funeral lamentations, ballads (patriotic songs, recitals of events, songs of travellers, etc.), songs of farewell, pastorals (melancholy in character). The poetic texts, singers and their instruments, and a comparaison of the Andian musical folklore as compared with that of America in general, and that of Spain, are also considered.

In the fourth part are given 204 songs with the melodies, grouped by subjects : religious chants, lamentations, etc. In each case the name of place from which the song came is recorded.

The work is sumptuous, as one might expect of a product of the GEUTHNER press, the illustrations especially being most beautifully reproduced. It is a valuable work for anyone interested in the Incan Empire or Indian music in America. A new edition should contain a reference to the work which Professor D. N. LEHMER, of the University of California has done in this connection (1).

R. C. A.

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(1) For example, the two publications at Berkeley, Cal., in 1927 : *Down the stream and other Indian Songs* (5 songs arranged for flute and voice with piano accompaniments ; *Two Indian Choruses : Buffalo Song and Invocation to the Sun*). The *Invocation* was based on the wonderful old Incan air, Kanmi Deos Kanki, found in D'HARCOURT's book.

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